Exercise 7.16

1. Write down the equations of the asymptotes of each of the following curves.

(a) $y = \dfrac{2}{1-x}$ (b) $y = \dfrac{x}{1-x}$ (c) $y = \dfrac{x+1}{x^2}$

(d) $y = \dfrac{x-3}{x-4}$ (e) $y = \dfrac{x}{2x+3}$

2. Determine the behaviour of y as $x \to \infty$ and $x \to -\infty$ if

(a) $y = \dfrac{x}{1+x^2}$ (b) $y = x^2e^{-x}$ (c) $y = x \ln x$

(d) $y = x^3 + x - 3$ (e) $y^2 = 4ax$

3. Sketch the graphs of the following functions, showing clearly any asymptotes, turning points, points of intersection with the axes and the behaviour of the curve when x and/or y are very large.

(a) $y = \dfrac{2}{1-x}$ (b) $y = \dfrac{x}{1-x}$ (c) $y = \dfrac{x-3}{x-4}$

(d) $y = x^2e^x$ (e) $y = x^3 + 2x^2 - x - 2$ (f) $y = \dfrac{3x}{x-4}$

(g) $y = \dfrac{e^x}{x}$

4. Find the turning points on the curve with the equation $y = 4e^{4x} + 9e^{-x}$. Hence, sketch the curve.

5. Find the x-coordinate of the point on the curve

$$y = \frac{\ln x}{x^2} \quad (x > 0)$$

for which $\dfrac{dy}{dx} = 0$ and determine whether it is a maximum or minimum point. Sketch the curve for $x > 0$.

[You may assume that $y \to 0$ as $x \to \infty$]

6. For the curve with the equation $y^2 = x(2-x)^2$,

(a) state the axis of symmetry of the curve,

(b) show that there is no point on the curve for which x is negative,

(c) find the coordinates of the points on the curve at which the tangents are parallel to the x-axis.

(d) sketch the curve.

7. Sketch the graph of the following functions on separate diagrams.

(a) $y = x^2 + 2$ (b) $y = \dfrac{1}{x^2+2}$

State the coordinates of the turning points for both curves.

For (b), show the behaviour of the curve for large positive values of x and large negative values of y.

8. (a) Find the equations of the asymptotes of the curve whose equation is $y = \dfrac{x+2}{x-3}$.

(b) Find the points of intersection of the curve with the coordinate axes. Find also the stationary points of the curve.

(c) Sketch the curve.

9. If $y = \dfrac{3(x-2)}{x(x+6)}$, find $\dfrac{dy}{dx}$ and deduce the values of x when $\dfrac{dy}{dx} = 0$. Determine the nature of these points. Sketch the graph showing the above properties and the asymptotes.

10. Find the coordinates of the turning point on the curve $y = e^x + 2e^{-x}$ and show that it is a minimum turning point. Sketch the curve.

11. Given that $f(x) = 6x^2 + x - 12$, find the minimum value of $f(x)$ and the values of x for which $f(x) = 0$.

Using the same axes, sketch the curves $y = f(x)$ and $y = \frac{1}{f(x)}$, labelling each curve clearly.

12. Given that $y = \frac{x-3}{x-4}$,

(a) find $\frac{dy}{dx}$,

(b) find the equation of the tangent to the curve at the point (6, 1.5),

(c) find the equation of the normal to the curve at the point (5, 2),

(d) use your answer from (a) to deduce that the curve has no turning points and sketch the graph.

13. For the curve $y = x^p e^{-\frac{1}{2}x}$, where p is an integer greater than 1, find the value of p when

(a) $\frac{dy}{dx} = 0$, (b) $\frac{d^2y}{dx^2} = 0$.

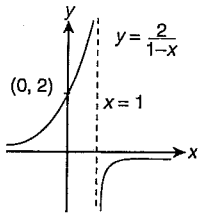
Sketch the graphs of the functions $y = x^2 e^{-\frac{1}{2}x}$ and $y = x^3 e^{-\frac{1}{2}x}$, showing clearly the turning point, point of inflexion and the behaviour of the curve when $x \to \pm\infty$.

Exercise 7.16

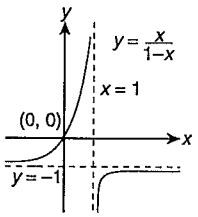
1. (a) $x = 1$, $y = 0$ (b) $x = 1$, $y = -1$
(c) $x = 0$, $y = 0$ (d) $x = 4$, $y = 1$
(e) $x = -\frac{3}{2}$, $y = \frac{1}{2}$

2. (a) $x \to \infty$, $y \to 0$; $x \to -\infty$, $y \to 0$
(b) $x \to \infty$, $y \to 0$; $x \to -\infty$, $y \to \infty$
(c) $x \to \infty$, $y \to \infty$; $x \to -\infty$
(d) $x \to \infty$, $y \to \infty$; $x \to -\infty$, $y \to -\infty$
(e) $x \to \infty$, $y \to \pm\infty$; $x \to -\infty$

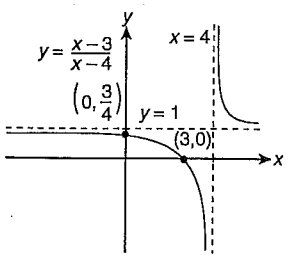
3. (a)



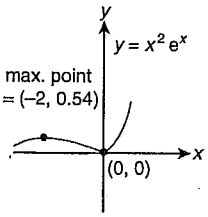
(b)



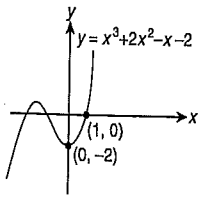
(c)



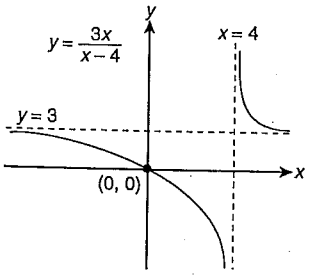
(d)



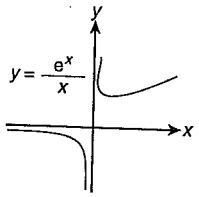
(e)



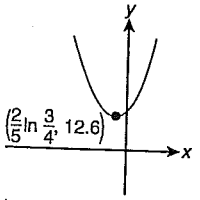
(f)



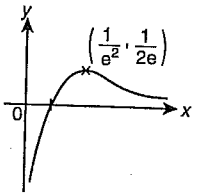
(g)



4. $(\frac{2}{5}\ln\frac{3}{4}, 12.6)$



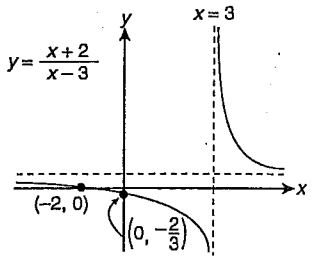
5. $x = e^{\frac{1}{2}}$ maximum



8. (a) $y = 1$, $x = 3$

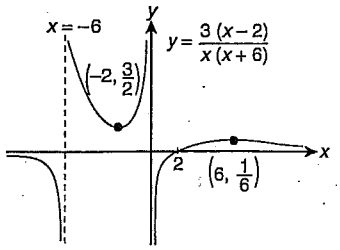
(b) $(-2, 0)$, $(0, -\frac{2}{3})$; No stationary point

(c)

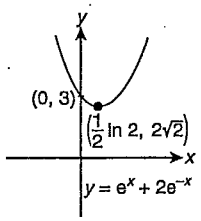


9. $-\frac{3(x+2)(x-6)}{x^2(x+6)^2}$; $x = -2, 6$

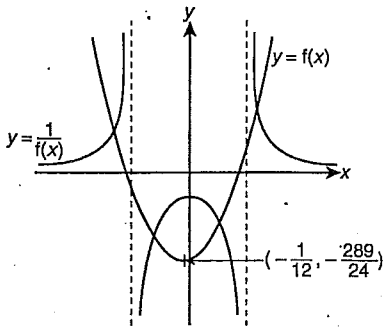
$(-2, \frac{3}{2})$ minimum, $(6, \frac{1}{6})$ maximum



10. $(\frac{1}{2}\ln 2,\ 2\sqrt{2})$



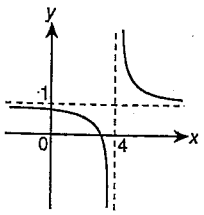
11. $-\frac{289}{24}$, $-\frac{3}{2}$, $\frac{4}{3}$



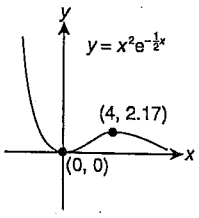
12. (a) $-\frac{1}{(x-4)^2}$ (b) $4y + x = 12$

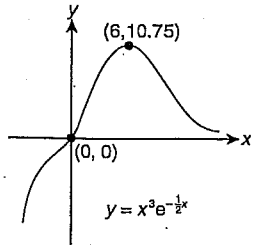
(c) $y = x - 3$

(d)



13. (a) $x = 0$, $x = 2p$ (b) $x = 0$ and $2(p \pm \sqrt{p})$

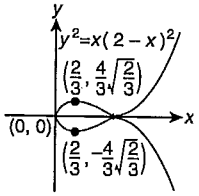




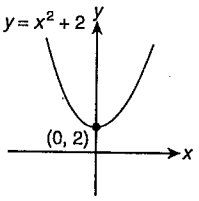
6. (a) x-axis

(c) $(\frac{2}{3}, \frac{4}{3}\sqrt{\frac{2}{3}})$, $(\frac{2}{3}, -\frac{4}{3}\sqrt{\frac{2}{3}})$

(d)



7. (a)



(b)