# Differentiating Parametric Functions

## Quick Review 7.4

**1** Find $\frac{dy}{dx}$ and leave answers in terms of $t$:

(a) $x = \frac{2t}{1+t^2}, y = \frac{1-t^2}{1+t^2}$

(b) $x = \frac{1}{\sqrt{1+t^2}}, y = \frac{t}{\sqrt{1+t^2}}$

(c) $x = \frac{t^2+1}{2t}, y = \frac{t^2-1}{2t}$

(d) $x = \frac{t}{1-t}, y = \frac{t^2}{1-t}$

**2** Find $\frac{dy}{dx}$, and express your answers in terms of $x$ where possible:

(a) $x = \frac{2t}{t+2}, y = \frac{3t}{t+3}$

(b) $x = t^2, y = t^3$

(c) $x = 1 + t, y = 3t^2 - 1$

(d) $x = t^2, y = \frac{t^2+1}{t}$

**3** Find $\frac{dy}{dx}$, and express your answers in terms of $x$ where possible:

(a) $x = \sin t, y = 1 + \cos^2 t$

(b) $x = a \sec t, y = b \tan t$

(c) $x = a \cos t, y = b \sin t$

**4** Find the equations of the tangents and normals to the following curves at the points indicated:

(a) $x = (t-2)(t+2), y = t(t-2)(t+2)$, $(-3, -3)$

(b) $x = t^2, y = t^3$, $(1, -1)$

(c) $x = 3 \cos t, y = 2 \sin t$, $(\frac{3}{2}, \sqrt{3})$

(d) $x = 4t^3, y = 3t^4$, $(4, 3)$

**5** Find the equations of the tangents and normals to the following curves at the points whose parameter is $t$:

(a) $x = \frac{a}{\cos t}, y = b \tan t$

(b) $x = t^3, y = 3t^2$

(c) $x = 2t, y = t^2$

(d) $x = a \cos t, y = b \sin t$

**6** Show that the tangent to the curve

$$y = -2at^3, x = 3at^2 + 2a$$

at the point with parameter $t$ is also the normal to the curve $y = 2at, x = at^2$ at the point with parameter $t$.

**7** Show that the tangent at the point $t$ on the curve $x = a \cos^3 t, y = a \sin^3 t$ is the line

$$y \cos t + x \sin t = a \sin t \cos t$$

Show that the tangent meets the axes in points whose distance apart is $a$.

**8** Find the equation of the tangent and normal to the curve

$$x = t^3 - t^2, y = t^2 - 1$$

at the point $(4, 3)$.

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**Quick Review 7.4**

**1** (a) $-\frac{2t}{1-t^2}$ (c) $\frac{t^2+1}{t^2-1}$

(b) $-\frac{1}{t}$ (d) $2t - t^2$

**2** (a) $\frac{y^2}{x^2}$ (c) $6(x-1)$

(b) $\frac{3}{2}\sqrt{x}$ (d) $\frac{x-1}{2x^{\frac{3}{2}}}$

**3** (a) $-2x$ (c) $-\frac{b^2x}{a^2y}$

(b) $\frac{bx}{a\sqrt{x^2-a^2}}$

**4** (a) $2y + x + 9 = 0, 2x - y + 3 = 0$

(b) $2y + 3x = 1, 22x - 3y = 5$

(c) $3\sqrt{3}y + 2x = 12, 6\sqrt{3}x - 4y = 5\sqrt{3}$

(d) $y - x + 1 = 0, y + x = 7$

**5** (a) $bx \sec t - ay \tan t = ab$
$ax \sin t + by = (a^2 + b^2) \tan t$

(b) $ty - 2x = t^3, 2y + tx = 6t^2 + t^4$

(c) $y - tx + t^2 = 0, ty + x = t(t^2 + 2)$

(d) $bx \cos t + ay \sin t = ab$
$ax \sin t - by \cos t = (a^2 - b^2) \sin t \cos t$

**6** $y + tx = 2at + at^3$

**8** $x - 2y + 2 = 0, 2x + y = 11$