

DIFFERENTIATION

Quick Review 7.2

- 1 Find the gradient function $\frac{dy}{dx}$ for each of the following:
- $y = 2x^2 + x + 1$
 - $y = 2x^3 - 4x + 7$
 - $y = 5x^2 - 6x^3 + 3$
 - $y = 3x^4 + 2x^3 - 5x$
- 2 Find the gradient function $\frac{dy}{dx}$ for each of the following:
- $y = 2e^{x^2} + 3x^2 + x^3$
 - $y = 2e^{3x} + 3e^{2x} + 2x$
 - $y = 4e^{3x+1} - 2x + 1$
 - $y = 3e^{x^3} + 4e^{2x^2+1} + 4$
- 3 Differentiate each of the following with respect to x :
- $y = x + e^x + \log_e(2x + 1)$
 - $y = \log_e [x(x + 5)(x + 3)]$
 - $y = \log_e \left[\frac{(x + 2)^4}{(x - 2)} \right] + 2e^x$
 - $y = 5x^2 + 2e^{\sin x} + \log(x^2 + x - 4)$
- 4 Differentiate each of the following with respect to x :
- $y = 3 \sin 2x + \cos 3x - \tan 5x$
 - $y = 2 \tan \left(\frac{x}{5} + \frac{\pi}{2} \right) + \cos \left(\frac{\pi}{4} - \frac{x}{2} \right) + \sin \left(x + \frac{\pi}{2} \right)$
 - $y = \sin(x^2 + 2) + 4e^{\cos 2x} + 1$
 - $y = \cot(2x + 1) + 2e^{\tan 2x} + \log_e \sin x$
- 5 Differentiate each of the following functions with respect to x :
- $y = x^2 e^x + \sqrt{x} \log_e x + x^2 \sin x + 1$
 - $y = (x^4 + 3x^2 + 2x) \log_e x$
 - $y = (x^2 - 3 \ln x)(2e^x + 3x)$
 - $y = 2x^2 \sin 3x + x^2 \log_e x + 3$
- 6 Differentiate each of the following functions with respect to x :
- $y = \frac{1 + x^2}{(1 + x)^2}$
 - $y = \frac{x^2(2 - x)}{\sqrt{x^2 - 1}}$
 - $y = \frac{3x - 5}{5x - 7}$
 - $y = \frac{(5x - 3)^2}{3x + 1}$
- 7 Differentiate each of the following functions with respect to x :
- $y = 4(3x^2 + 5)^3$
 - $y = [^3\sqrt{x^2 + 1}]^4$
 - $y = \frac{2}{(7x - 3)^2}$
 - $y = \frac{1}{(x^3 + x - 5)^4}$
- 8 Find the gradient function $\frac{dy}{dx}$ for each of the following:
- $y = (x^3 + 5)^{10} \log_e(x^3 + 5)$
 - $y = \frac{\log_e x}{\sqrt{x^2 + 1}}$
 - $y = (1 + x^2)^8 e^x$
 - $y = \frac{1 + e^x}{(3 + x)^3}$
 - $y = \frac{e^x}{\sqrt{x^2 + 1}}$
- 9 Differentiate each of the following function with respect to x :
- $y = \sin^5 2x + e^x \cos 3x$
 - $y = e^{\tan x} + \log_e \tan x$
 - $y = \sin^2(x + 1) + \cos^3(3x)$
 - $y = \cos^2(2x^2 + 1) + \tan^3(x^2 + 1)$
 - $y = \cos^4(\pi x) \sin^3(\pi x)$
- 10 Use the technique of logarithmic differentiation to differentiate the following functions with respect to x :
- $y = \sqrt{\frac{(1 + x^2)^3}{2 + x^2}}$
 - $y = \frac{(1 + x)}{\sqrt{1 - 3x}}$
 - $y = \frac{(1 - 2x)^3}{(x + 2)^4}$
 - $y = \frac{(1 - \sqrt{x})^2}{\sqrt{x^2 - 1}}$
 - $y = (1 - x^2) \sqrt[3]{1 - 2x}$

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- 1 (a) $4x + 1$ (c) $10x - 18x^2$
 (b) $6x^2 - 4$ (d) $12x^3 + 6x^2 - 5$
- 2 (a) $4xe^{x^2} + 6x + 3x^2$ (c) $12e^{3x+1} - 2$
 (b) $6e^{3x} + 6e^{2x} + 2$ (d) $9x^2e^{x^3} + 16xe^{2x^2} + 1$
- 3 (a) $1 + e^x + \frac{2}{2x+1}$
 (b) $\frac{1}{x} + \frac{1}{x+5} + \frac{1}{x+3}$
 (c) $\frac{4}{x+4} - \frac{1}{x-2} + e^{2x}$
 (d) $10x + 2 \cos x e^{\sin x} + \frac{2x+1}{x^2+x-4}$
- 4 (a) $6 \cos 2x - 3 \sin 3x - 5 \sec^2 5x$
 (b) $\frac{2}{5} \sec^2 \left(\frac{x}{5} + \frac{\pi}{2} \right) + \frac{1}{2} \sin \left(\frac{\pi}{4} - \frac{x}{2} \right) + \cos \left(x + \frac{\pi}{2} \right)$
 (c) $2x \cos(x^2 + 2) - 8 \sin 2x e^{\cos 2x}$
 (d) $-2 \operatorname{cosec}^2(2x + 1) + 4 \sec^2 2x e^{\tan 2x} + \cot x$
- 5 (a) $(2x + x^2) e^x + \frac{1}{2\sqrt{x}} (\log_e x + 2) + 2x \sin x + x^2 \cos x$
 (b) $x^3 + 3x + 2 + (4x^3 + 6x + 2) \log_e x$
 (c) $(2x^2 + 4x - \frac{6}{x} - 6 \log_e x) e^x + 9x^2 - 9 \log_e x - 9$
 (d) $4x \sin 3x + 6x^2 \cos 3x + 2x \log_e x + x$
- 6 (a) $\frac{2(x-1)}{(x+1)^3}$ (c) $\frac{4}{(5x-2)^2}$
 (b) $\frac{(-2x^3 + 2x^2 + 3x - 4)x}{\sqrt{(x^2-1)^3}}$ (d) $\frac{(5x-3)(15x+19)}{(3x+1)^2}$
- 7 (a) $72x(3x^2 + 5)^2$ (c) $-\frac{28}{(7x-3)^3}$
 (b) $\frac{8x^3\sqrt{x^2+1}}{3}$ (d) $\frac{-4(3x^2+1)}{(x^3+x-5)^5}$
- 8 (a) $3x^2(x^3 + 5)^9 [1 + 10 \log_e(x^3 + 5)]$
 (b) $\frac{x^2(1 - \log_e x) + 1}{x(x^2 + 1)^{\frac{3}{2}}}$
 (c) $e^x(1 + x^2)^7 (x^2 + 16x + 1)$
 (d) $\frac{xe^x - 3}{(x+3)^4}$
 (e) $\frac{(x^2 - x + 1) e^x}{x(x^2 + 1)^{\frac{3}{2}}}$
- 9 (a) $10 \cos 2x \sin^4 2x + e^x(\cos 3x - 3 \sin 3x)$
 (b) $\sec^2 x e^{\tan x} + \frac{1}{\sin x \cos x}$
 (c) $2 \sin(x+1) \cos(x+1) - 9 \cos^2 3x \sin 3x$
 (d) $-8x \cos(2x^2 + 1) \sin(2x^2 + 1) + 6x \tan^2(x^2 + 1) \sec^2(x^2 + 1)$
 (e) $n\pi \cos^2(n\pi x) \sin^2(n\pi x) [3 \cos^2(n\pi x) - 4 \sin^2(n\pi x)]$
- 10 (a) $-\frac{x(2x^2 + 5)\sqrt{1+x^2}}{\sqrt{(2+x^2)^3}}$ (d) $\frac{(1-\sqrt{x})(1-x\sqrt{x})}{\sqrt{x(x^2-1)^{\frac{3}{2}}}}$
 (b) $\frac{5-3x}{2(1-3x)^{\frac{3}{2}}}$ (e) $\frac{14x^2-6x-2}{3\sqrt[3]{(1-2x)^2}}$
 (c) $\frac{-2(1-2x)^2(8-x)}{(x+2)^5}$