Exercise 5.6

1. Sketch the curves represented by each of the following equations. State the length of the major and minor axes in each case.

(a) $4x^2 + 9y^2 = 16$

(b) $8x^2 + 4y^2 = 9$

(c) $\dfrac{4x^2}{25} + \dfrac{4y^2}{9} = 1$

(d) $\dfrac{(x-2)^2}{4} + \dfrac{y^2}{8} = 1$

(e) $\dfrac{(x-1)^2}{16} + \dfrac{(y-3)^2}{9} = 1$

(f) $4x^2 + 9y^2 - 12x + 36y + 9 = 0$

2. Show that the following lines are tangents to the given ellipse, and find their points of contact.

(a) $x + 2y = 4$; $x^2 + 4y^2 = 8$

(b) $2x + y = 8$; $\dfrac{x^2}{12} + \dfrac{y^2}{16} = 1$

(c) $5x + 3y = 28$; $5x^2 + y^2 = 56$

(d) $3x + 7y = 13$; $3x^2 + 14y^2 = 26$

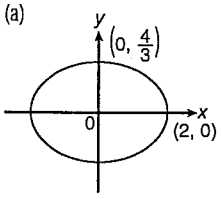
3. Prove that if the line $lx + my + n = 0$ touches the ellipse $b^2x^2 + a^2y^2 = a^2b^2$, then $a^2l^2 + b^2m^2 = n^2$.

4. Find the gradients of the tangents drawn from the point (4, 6) to the ellipse $x^2 + 12y^2 = 48$. Hence, find the equations of the tangents and their points of contact with the ellipse.

5. If m is the gradient of the tangent from the point (3, 2) to the ellipse $9x^2 + 16y^2 = 144$, find a quadratic equation in m. By noting whether the roots of this equation are real or imaginary, determine if the point (3, 2) lies within the ellipse.

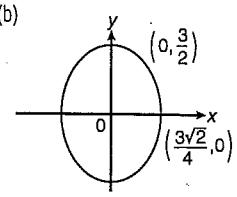
Exercise 5.6

1. (a)



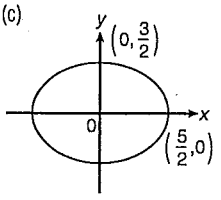
Major axis: 4
Minor axis: $\frac{8}{3}$

(b)



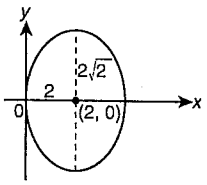
Major axis: 3
Minor axis: $\frac{3}{2}\sqrt{2}$

(c)



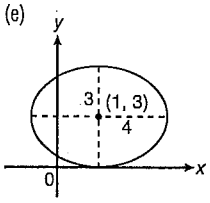
Major axis: 5
Minor axis: 3

(d)



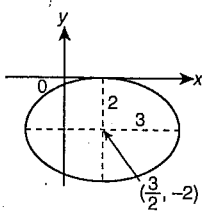
Major axis: $4\sqrt{2}$
Minor axis: 4

(e)



Major axis: 8
Minor axis: 6

(f)



Major axis: 6
Minor axis: 4

2. (a) (2, 1) (b) (3, 2)
(c) (2, 6) (d) (2, 1)

3. $\frac{1}{2}$, -2
$2y = x + 8$; $(-6, 1)$
$y + 2x = 14$; $(\frac{48}{7}, -14)$

4. $\frac{1}{2}$, -2; $2y = x + 8$, $(-6, 1)$;
$y + 2x = 14$; $(\frac{48}{7}, \frac{2}{7})$

5. $7m^2 + 12m + 5 = 0$; outside