Equations of Tangent and Normal to a curve

Quick Review 7.6 (a)

1 The gradient function of a curve is given by

$$\lim_{h\to 0}\left[\frac{\frac{1}{x+h}-\frac{1}{x}}{h}\right]$$

Find the equation of the curve and find the equation of the tangent to this curve at the point $(2, \frac{1}{2})$.

2 The gradient function of a curve is given by

$$\lim_{h\to 0}\left[\frac{(x+h)^3-x^3}{h}\right]$$

Find the equation of the curve and find the equation of the tangent to this curve at the point when $x = 2$.

3 Find the equation of the tangent and of the normal to the curves with the following equations and at the points or conditions indicated.
(a) $y = x^3 - 3x + 2$ at $(0, 2)$
(b) $y = 3x^3 - 7x^2 + 2x$, where $x = 2$
(c) $y = 2x^2 - 4x + 1$, where the gradient is 4
(d) $y = \log_e (x - 1)$, where $x = 2$
(e) $y = \cos 2x$, where $x = \frac{\pi}{4}$

4 Find the equation of the tangent and of the normal to the curves with the following equations and at the points or conditions indicated.
(a) $y^3 + x^3 + 3xy = 1$, where $x = 2$
(b) $y + x \cos y = \frac{\pi}{3}$, where $x = 0$
(c) $xy + x.\log_e(2 + y) = 1$, where $y = -1$
(d) $y + e^{-x} \cos x = 2e^{-x}$, where $x = 0$
(e) $x = t(t^2 + 1)$, $y = t^2 + 1$, where $t = p$

5 Find the equations of the tangents to the curve $xy = 6$ which are parallel to the line $4y + 6x = 5$.

6 Find the coordinates of the points of intersection of the curves $y^2 = x$ and $x^2 = y$. What are the equations of the tangents to the curves at these points?

7 Find the equation of the normal to the curve $4y = x^2$ at the point $(4, 4)$. Find also the coordinates of the point at which this normal meets the curve again, and show that the length of the chord so formed is $5\sqrt{5}$.

8 Find the points of intersection of the curves $y^2 = x$ and $x^2 = 8y$. Find also the gradient of the curves at these points of intersection, and hence find the angles at which the curves cut.

9 The graph of the curve $y = ax^3 + bx^2 + cx + d$ touches the x-axis at $x = -2$ and cuts the y-axis at $y = 5$ with a gradient of 3. Find
(a) a, b, c, d.
(b) the point at which the graph cuts the x-axis.
(c) the equation of the normal at this point.

10 The graph of the curve $y = ax^2 + bx + c$ cuts the line $y = x + 1$ at right angles at the point $(-2, -1)$, and cuts the y-axis at $y = -2$. Find the values of a, b and c.

11 Find the equations of the tangent and normal to the curve

$$x = 2t,\ y = t^2$$

at the point where $t = a$.
If the tangent and normal meet the y-axis in P and Q respectively, prove that

$$PQ = 2(1 + a^2).$$

12 The tangent at the point $P(ap^2, 2ap)$ on the curve: $x = at^2$, $y = 2at$, meets the axis of y in Q. Show that if the coordinates of the midpoint of PQ are (α, β), then

$$\alpha = \frac{1}{2}ap^2,\ \beta = \frac{3ap}{2}$$

Hence, or otherwise, find the locus of the midpoint of PQ.

Quick Review 7.6 (a)

1 $f(x) = \frac{1}{x}$

tangent : $4y + x = 4$

2 $f(x) = x^3$

tangent : $y = 12x - 6$

3 (a) $y + 3x = 2,\ 3y - x = 6$

(b) $y = 10x - 20,\ x + 10y = 2$

(c) $y = 4x - 7,\ x + 4y = 6$

(d) $y = x - 2,\ y + x = 2$

(e) $y + 2x = \frac{\pi}{2},\ 2y - x + \frac{\pi}{4} = 0$

4 (a) $y + x = 1,\ y - x + 3 = 0$

(b) $2y + x = \frac{2\pi}{3},\ y - 2x = \frac{\pi}{3}$

(c) $2y + x + 3 = 0,\ y = 2x + 1$

(d) $y + x = 1,\ y = x + 1$

(e) $(3p^2 + 1)\, y = 2px + (p^2 + 1)^2$

$2py + (3p^2 + 1)x = 3p(p^2 + 1)^2$

5 $3x + 2y - 12 = 0,\ 3x + 2y + 12 = 0$

6 $(0, 0), (1, 1);\ x = 0,\ 2y = x + 1$

$y = 0,\ y = 2x - 1$

7 $2y + x = 12,\ (-6, 9)$

8 Point (0, 0); gradient α and 0; angle = 90°

Point (4, 2), gradient $\frac{1}{4}$ and 1, angle = 30° 58′

9 (a) $-\frac{1}{2}, -\frac{3}{4}, 3, 5$

(b) (2.5, 0)

(c) $8x - 81y = 20$

10 $\frac{1}{4}, 0, -2$

11 $y - ax + a^2 = 0,\ ay + x - a(a^2 + 2) = 0$

12 $y^2 = \frac{9ax}{2}$