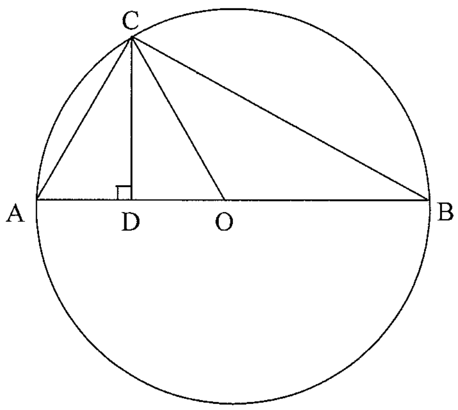
Topic 18: Exercises on Harder 3 Unit Circles & Triangles
Level 1 & 3

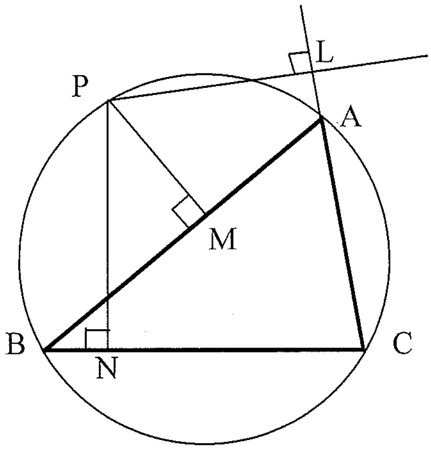
1. ADB is a straight line with $AD = a$ and $DB = b$. A circle is drawn on AB as diameter. DC is drawn perpendicular to AB to meet this circle at C.

(i) Show that $\Delta ADC \parallel\!| \Delta CDB$, and hence show that $DC = \sqrt{ab}$.

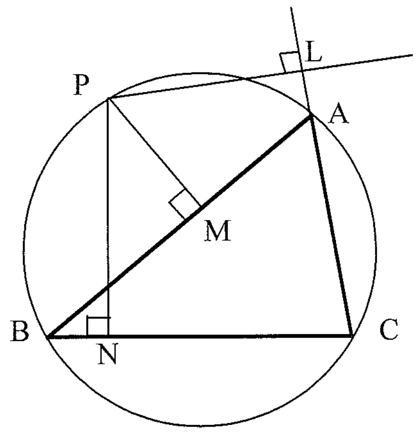
(ii) Deduce geometrically that if $a > 0$ and $b > 0$, then $\sqrt{ab} \leq \dfrac{a+b}{2}$.



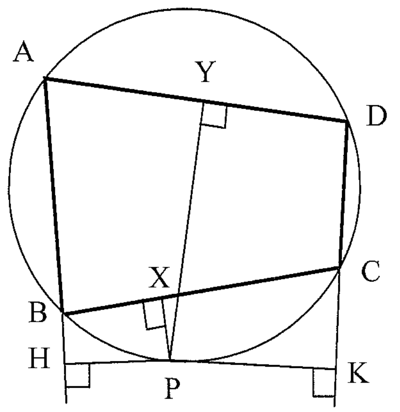
2. ABC is a triangle inscribed in the a circle. P is a point on the minor *arc* AB. The points L, M, and N are the feet of the perpendiculars from P to CA produced, AB, and BC respectively. Show that L, M and N are collinear. (The line NL is called the Simpson line.)



3. ΔABC is inscribed in a circle. P is a point on a minor *arc* AB. The points M, L and N are the feet of the perpendiculars from P to AB produced, AC and BC respectively. Show that ΔPNL ||| ΔPBA. Hint: use the fact that the points N, M, and L are collinear.



4. *ABCD* is a cyclic quadrilateral. *P* is a point on the circle through *ABC* and D. *PH*, *PX*, *PK* and *PY* are the perpendiculars from *P* to *AB* produced, *BC*, *DC* produced and *DA*, respectively.

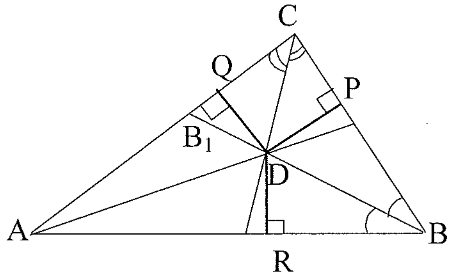


(i) Show that $\Delta XPK \ ||| \ \Delta HPY$

(ii) Hence show that $PX \cdot PY = PH \cdot PK$ and $\dfrac{PX \cdot PK}{PH \cdot PY} = \dfrac{(XK)^2}{(HY)^2}$.

Hint: Use the result of the Question 3.

5. ABC is a triangle. The internal bisectors of $\hat{B}$ and $\hat{C}$ meet at D. DP, DQ and DP are the perpendiculars from D to BC, CA and AB respectively. Show that $DR = DQ$ and deduce that the internal bisectors of the three angles of a triangle are concurrent.



6. ABC is a triangle. E and F are the midpoints of AC and AB respectively. BE and CF intersects at D. Show that the triangle DEF and DBS are similar and hence deduce that the three medians of a triangle are concurrent.