Exercise 6.9

1. If $\sin^{-1} x = \frac{2\pi}{5}$, find the value of $\cos^{-1} x$.

2. If $2\cos^{-1} x = \sin^{-1} x$, find the value of x.

3. Without using tables or calculators, show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$, where the angles are acute.

4. Solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1 - x)$.

5. Assuming that the angles are acute, solve the equation $\tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 2$.

6. Find, correct to two decimal places, the positive value of x which satisfies the equation $\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}$.

7. Solve the equation $\sin^{-1}(2x) = \frac{1}{3}\pi - \sin^{-1} x$.

8. Without using tables or calculators, show that $\cos^{-1}(\frac{3}{5}) - \tan^{-1}(-\frac{3}{4}) = \frac{\pi}{2}$.

9. Without using tables or calculators, evaluate
   - (a) $\sin[\tan^{-1}(-\frac{5}{12})]$
   - (b) $\sin^{-1}(\tan\frac{3\pi}{4})$
   - (c) $\tan[\cos^{-1}(\frac{5}{6})]$
   - (d) $\cos[2\cos^{-1}(-\frac{1}{2})]$
   - (e) $\sin[\frac{\pi}{3} + \cos^{-1}(\frac{1}{4})]$
   - (f) $\tan[\tan^{-1}(\frac{2}{3}) - \tan^{-1}(\frac{1}{2})]$

10. Prove that $\cos^{-1} x + \cos^{-1} y = \cos^{-1}[xy - \sqrt{(1 - x^2)(1 - y^2)}]$.

Exercise 6.9

1. $\frac{\pi}{10}$ **2.** $\frac{\sqrt{3}}{2}$

4. $\frac{\sqrt{17} - 1}{4}$ **5.** $\frac{-2 \pm \sqrt{6}}{2}$

6. 0.28 **7.** $\frac{1}{2}\sqrt{\frac{3}{7}}$

9. (a) $-\frac{5}{13}$ (b) $-\frac{\pi}{2}$ (c) $\frac{\sqrt{11}}{5}$

(d) $-\frac{1}{2}$ (e) $\frac{\sqrt{3} + \sqrt{15}}{8}$ (f) $\frac{1}{8}$