Higher Derivatives

Quick Review 7.5

1 Find the first and second derivatives of the following:

(a) $y = 4x^5 + x^2 + 8x + 5$

(b) $y = \dfrac{1}{x+1}$

(c) $y = \sqrt{2x+1}$

(d) $y = 2x^2 + 3x - 2 + \dfrac{1}{x}$

2 Find the first and second derivatives of the following:

(a) $\sin \pi t$

(b) $3 \sin^4 (\omega t)$

(c) $e^{\sin \omega t}$

(d) $\tan (\omega t - \frac{\pi}{2})$

3 Find the first and second derivatives of the following:

(a) $e^t \log_e t$

(b) $e^{2\omega t} \cos t$

(c) $te^{\sin t}$

(d) $\dfrac{\log_e t}{t}$

4 Use implicit differentiation to find the first and second derivative of the following:

(a) $y^3 = xy + 1$

(b) $y = \dfrac{1}{x+y}$

(c) $y = x - \dfrac{1}{x+y}$

(d) $x^2 + 2xy + 3y^2 = 2$

5 Find the rate of change with respect to x of the slope of the tangent line to the curve $x^2 + y^2 = xy + 3$ at the point $(1, 2)$ on the curve.

6 If y is a function of x given by:

$$x^3 + y^3 = 2xy + 8$$

find the values of $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at the point $(2, 2)$ on the curve.

7 If $y - 2e^{-2x} \sin x = 0$, prove that

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

8 If $ve^{kt} = A \sin \omega t + B \cos \omega t$, A, B being arbitrary constants and k, ω given fixed constants, prove that

$$\frac{d^2v}{dt^2} + 2k\frac{dv}{dt} + v(\omega^2 + k^2) = 0$$

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1 (a) $20x^4 + 2x + 8$, $80x^3 + 2$

(b) $-\dfrac{1}{(x+1)^2}$, $\dfrac{2}{(x+1)^3}$

(c) $\dfrac{1}{\sqrt{2x+1}}$, $-\dfrac{1}{(2x+1)^{\frac{3}{2}}}$

(d) $4x + 3 - \dfrac{1}{x^2}$, $4 + \dfrac{2}{x^3}$

2 (a) $\pi \cos \pi t$, $-\pi^2 \sin \pi t$

(b) $12w \sin^3 wt \cos wt$

$12w^2 \sin^2 wt\,[3 \cos^2 wt - \sin^2 wt]$

(c) $w \cos wt\, e^{\sin wt}$, $w^2(\cos^2 wt - \sin wt)e^{\sin wt}$

(d) $w \sec^2 (wt - \frac{\pi}{2})$

$-2w^2 \sin(wt - \frac{\pi}{2}) \cos^3 (wt - \frac{\pi}{2})$

3 (a) $e^t \log_e t + \dfrac{e^t}{t}$, $e^t \log_e t + \dfrac{e^t}{t} + \dfrac{e^t(t-1)}{t^2}$

(b) $we^{2wt}\,[2 \cos wt - \sin wt]$

$w^2e^{2wt}[3 \cos wt - 4 \sin wt]$

(c) $e^{\sin t}[1 + t \cos t]$

$e^{\sin t}[2 \cos t + t^2 \cos t - t \sin t]$

(d) $\dfrac{1}{t^2}[1 - \log_e t]$, $\dfrac{1}{t^3}[2 \log_e t - 3]$

4 (a) $\dfrac{y}{3y^2 - x}$, $\dfrac{-2xy}{(3y^2 - x)^3}$

(b) $-\dfrac{y}{x + 2y}$, $\dfrac{2}{(x + 2y)^3}$

(c) $\dfrac{x}{y}$, $-\dfrac{1}{y^3}$

(d) $-\dfrac{(x + y)}{(x + 3y)}$, $\dfrac{(3y - x)(x + y) - (x + 3y)^2}{(x + 3y)^3}$

5 $-\dfrac{2}{3}$

6 -1, $-\dfrac{7}{2}$