

Implicit Differentiation

Quick Review 7.3

- 1 Find $\frac{dy}{dx}$ for the following:
- (a) $x^4 - 2xy + y^5 = 28$
 (b) $x^2 \log_e y + xy = 4x + 8$
 (c) $x^2 \cos y + y \sin x = 8$
 (d) $\cos x \sin y + xe^{\cos y} = 18$
- 2 Find $\frac{dy}{dx}$ for the following:
- (a) $2x^2\sqrt{y} + y^3 + x^4 = 28$
 (b) $(x^3 + y)y^2 = 2x + 38$
 (c) $ye^x + xe^y = y$
 (d) $\cos(e^y) + \log_e(\sin y) = 28 - xy$
- 3 Differentiate implicitly the following:
- (a) $2x^4 - x^3y = 2x + y$
 (b) $x^2 \sin y - y \cos x = x^2y - 4$
 (c) $xy^2 + xy + y^3 = 5$
 (d) $2x^2 + y^2 + 3y = xy + 4$
- 4 Differentiate implicitly and find the gradient of the curve at the indicated point.
- (a) $x^2y^3 = 5xy^2 + 4y + 4$, (3, 2)
 (b) $xy + x + y^2 = 7$, (1, 2)
 (c) $x\sqrt{y} + x^2 = y^2 - 13$, (1, 4)
 (d) $x^2y + y^2 = 2x + 1$, (2, 1)
- 5 Use implicit differentiation to find the equation of the tangent line to the curve at the indicated point.
- (a) $y^2 + x^2 + y^3 = 3xy + 7$, (1, 2)
 (b) $4xy^2 + y^4 = -98 + 2x^4$, (3, 2)
 (c) $y(y^2 + 7x) = x^3$, (4, 2)
 (d) $7y^2 - 4x^2 + 36 = 0$, (4, 2)
- 6 Use implicit differentiation to find the equation of the normal line at the indicated point:
- (a) $xy + 2x + y = 6$, (1, 2)
 (b) $x^2y - 2x + y^3 = 1$, (2, 1)
 (c) $x^2 + y^2 + xy^3 = 4$, (0, 2)
 (d) $x^2y^2 + xy^4 = 2$, (1, 1)
 (e) $2x^3y + 2y^4 = 2 + x^4$, (2, 1)
 (f) $y\sqrt{x} = 12 + x\sqrt{y}$, (9, 16)

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- 1 (a) $\frac{2y^3 - 4x^3}{5y^4 - 6xy^2}$ (c) $\frac{2x \cos y + y \cos x}{x^2 \sin y - \sin x}$
 (b) $\frac{y(4 - y - 2x \log_e y)}{x^2 + xy}$ (d) $\frac{\sin x \sin y - e^{\cos y}}{\sin x \cos y - x \sin y e^{\cos y}}$
- 2 (a) $\frac{-4(xy + x^3\sqrt{y})}{x^2 + 3y^2}$ (c) $\frac{ye^x + e^y}{(1 - e^x - xe^y)}$
 (b) $\frac{2 - 3x^2y^2}{2x^3y + 3y^2}$ (d) $\frac{y}{e^y \sin e^y - x - \cot y}$
- 3 (a) $\frac{8x^3 - 3x^2y - 2}{1 + x^2}$ (c) $\frac{-y(y+1)}{2xy + x + 3y^2}$
 (b) $\frac{2xy - 2x \sin y - y \sin x}{x^2 \cos y - \cos x - x^2}$ (d) $\frac{y - 4x}{2y - x + 3}$
- 4 (a) $-\frac{7}{11}$ (c) $\frac{16}{31}$
 (b) $-\frac{3}{5}$ (d) $-\frac{2}{7}$
- 5 (a) $13y = 4x + 22$ (c) $20y = 17x - 7$
 (b) $2y = 5x - 11$ (d) $7y = 8x - 18$
- 6 (a) $2y = x + 3$ (d) $y = 2x - 1$
 (b) $2y = 7x - 12$ (e) $y + 3x = 7$
 (c) $2y = x + 4$ (f) $32y + 45x = 917$