

Exercise 8.5

Express the functions in each of the following integrals in partial fractions and hence perform the integration.

1. $\int \frac{x}{x+2} dx$

7. $\int \frac{2x-4}{(x^2+4)(x+2)} dx$

2. $\int \frac{1}{(1-x)(3x-2)} dx$

8. $\int \frac{4x^2}{x(2x-1)^2} dx$

3. $\int \frac{2x+3}{(x-4)(5x+2)} dx$

9. $\int \frac{x^2}{x+1} dx$

4. $\int \frac{1}{x^2+4x+3} dx$

10. $\int \frac{x^2+x+5}{x(x+1)} dx$

5. $\int \frac{7x-2}{(x-2)^2(2x+3)} dx$

11. $\int \frac{x^2+2x+4}{(2x-1)(x^2-1)} dx$

6. $\int \frac{1}{x^2(1-x)} dx$

12. $\int \frac{12x}{(2-x)(3-x)(4-x)} dx$

13. Express $\frac{1}{x^2(x-1)}$ in the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$, where A , B and C are constants.

Hence, find $\int \frac{1}{x^2(x-1)} dx$.

14. By using the substitution $u = e^x$, show that

$$\int \frac{e^x - 1}{e^x + 1} dx = \int \frac{u - 1}{u(u + 1)} du.$$

Hence, by using partial fractions, find $\int \frac{e^x - 1}{e^x + 1} dx$.

Exercise 8.5

1. $x - 2 \ln |x + 2| + c$

2. $\ln \left| \frac{3x-2}{1-x} \right| + c$

3. $\frac{1}{2} \ln |x-4| - \frac{1}{10} \ln |5x+2| + c$

4. $\frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + c$

5. $\frac{25}{49} \ln \left| \frac{x-2}{2x+3} \right| - \frac{12}{7} \frac{1}{x-2}$

6. $\ln \left| \frac{x}{1-x} \right| - \frac{1}{x} + c$

7. $\ln \left| \frac{\sqrt{x^2+4}}{x+2} \right| + c$

8. $\ln |2x-1| - \frac{1}{2x-1} + c$

9. $\frac{x^2}{2} - x + \ln |x+1| + c$

10. $x + 5 \ln \left| \frac{x}{x+1} \right| + c$

11. $\frac{1}{2} \ln \left| \frac{(x-1)^2(x+1)}{(2x-1)^2} \right| + c$

12. $12 \ln \left| \frac{(3-x)^3}{(2-x)(4-x)^2} \right| + c$

13. $A = -1, B = -1, C = 1; \ln \left| \frac{x-1}{x} \right| + \frac{1}{x} + c$

14. $-x + 2 \ln |e^x + 1| + c$