

Exercise 8.3

1. Find the following integrals using the suggested substitution.

(a) $\int x(x^2 + 2)^3 \, dx, u = x^2 + 2$

(b) $\int 2x(x^2 - 4)^{\frac{1}{2}} \, dx, u = x^2 - 4$

(c) $\int x\sqrt{2x^2 - 5} \, dx, u = 2x^2 - 5$

(d) $\int \frac{x}{\sqrt{x+9}} \, dx, u = x + 9$

(e) $\int \frac{2x+1}{(x-3)^6} \, dx, u = x - 3$

(f) $\int \frac{3x}{\sqrt{4-x}} \, dx, u^2 = 4-x$

(g) $\int \sqrt{1-x^2} \, dx, x = \sin \theta$

(h) $\int \sin^5 x \, dx, u = \cos x$

(i) $\int \sin^4 x \cos x \, dx, u = \sin x$

(j) $\int \frac{1}{4+x^2} \, dx, x = 2 \tan \theta$

(k) $\int \frac{x^2+1}{\sqrt{x^3+3x+4}} \, dx, u = x^3 + 3x + 4$

(l) $\int \cos^3 x \sin x \, dx, u = \cos x$

(m) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx, u^2 = x$

(n) $\int \frac{1}{(9-x^2)^{\frac{3}{2}}} \, dx, x = 3 \sin \theta$

2. By using a suitable substitution, find the following integrals.

(a) $\int 2x(1-x)^7 \, dx$

(b) $\int \frac{x}{\sqrt{1-4x^2}} \, dx$

(c) $\int \frac{x+3}{(4-x)^5} \, dx$

(d) $\int \frac{1}{\sqrt{9-x^2}} \, dx$

(e) $\int \frac{x^2}{\sqrt{4-x^2}} \, dx$

(f) $\int \sin^3 x \, dx$

(g) $\int \sin^3 x \cos x \, dx$

Exercise 8.3

1. (a) $\frac{(x^2+2)^4}{8} + C$

(b) $\frac{2}{3}(x^2-4)^{\frac{3}{2}} + C$

(c) $\frac{1}{6}(2x^2-5)^{\frac{3}{2}} + C$

(d) $\frac{2}{3}(x+9)^{\frac{1}{2}}(x-18) + C$

(e) $\frac{1-5x}{10(x-3)^5} + C$

(f) $-2(8+x)\sqrt{4-x} + C$

(g) $\frac{1}{2}(\sin^{-1} x + x\sqrt{1-x^2}) + C$

(h) $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

(i) $\frac{1}{5}\sin^5 x + C$

(j) $\frac{1}{2}\tan^{-1} \frac{x}{2} + C$

(k) $\frac{2}{3}\sqrt{x^3+3x+4} + C$

(l) $-\frac{1}{4}\cos^4 x + C$

(m) $\frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + C$

(n) $\frac{x}{9\sqrt{9-x^2}} + C$

2. (a) $-\frac{1}{36}(8x+1)(1-x)^8$

(b) $-\frac{1}{4}\sqrt{1-4x^2} + C$

(c) $\frac{5+4x}{12(4-x)^4} + C$

(d) $\sin^{-1}(\frac{x}{3}) + C$

(e) $2\sin^{-1}(\frac{x}{2}) - \frac{1}{2}x\sqrt{4-x^2} + C$

(f) $-\cos x + \frac{1}{3}\cos^3 x + C$

(g) $\frac{\sin^4 x}{4} + C$