Revision Exercise

1. Find the following indefinite integrals.

(a) $\int \frac{1}{2x-9}\,dx$ (b) $\int \frac{2x+3}{3x^2+9x-1}\,dx$ (c) $\int \frac{e^{3x}+4}{e^{2x}}\,dx$

(d) $\int \frac{x}{\sqrt{2x-1}}\,dx$ (e) $\int \frac{x}{\sqrt{4-x^2}}\,dx$ (f) $\int x^2 e^{-x^3}\,dx$

(g) $\int \frac{x+1}{x(2x+1)}\,dx$ (h) $\int \frac{x(2x+1)}{x+1}\,dx$

2. Evaluate

(a) $\int_2^3 \frac{1}{x(x^2-1)}\,dx$ b) $\int_0^1 x(1-x)^{\frac{1}{2}}\,dx$ (c) $\int_0^1 x e^{-3x}\,dx$

(d) $\int_0^1 x\sqrt{1+x}\,dx$ (e) $\int_1^e (2x+1)\ln x\,dx$ (f) $\int_0^3 \frac{x}{1+x^2}\,dx$

(g) $\int_1^4 (\frac{3}{x}-\sqrt{x})^2\,dx$ (h) $\int_0^1 \frac{1-4x}{3+x-2x^2}\,dx$

3. By using the substitution $u^2 = 2x+1$, evaluate $\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx$

4. By means of a trigonometrical substitution, prove that

$$\int_0^1 \frac{2x+1}{\sqrt{(4-x^2)}}\,dx = 4-2\sqrt{3}+\frac{1}{6}\pi.$$

5. Show that $\frac{d}{dx}(\frac{x}{1+5x}) = \frac{1}{(1+5x)^2}$. Hence, evaluate $\int_1^3 (\frac{4}{1+5x})^2\,dx$.

6. Find $\frac{d}{dx}(x\cos x)$. Hence, evaluate

(a) $\int_0^{\pi} \cos x\,dx - \int_0^{\pi} x\sin x\,dx$ (b) $\int_2^{\pi} x\sin x\,dx$.

7. If $a > 1$ and $\int_1^a \frac{x^4-1}{x^3}\,dx = \frac{9}{8}$, find a.

8. Express $\frac{x-2}{2x^2-x-3}$ in partial fractions and hence evaluate $\int_2^3 \frac{x-2}{2x^2-x-3}\,dx$.

9. Evaluate the following integrals.

(a) $\int_0^1 \frac{8}{3+4x}\,dx$ (b) $\int_0^1 \frac{8}{\sqrt{(3+4x)}}\,dx$ (c) $\int_0^1 \frac{8x}{3+4x}\,dx$

10. (a) Show that $\int_1^2 \frac{(x-1)(5x+2)}{(2x-1)(x^2+2)}\,dx = \frac{1}{2}\ln\frac{8}{3}$.

(b) By using the substitution $x = \frac{1}{2}(1+\sin\theta)$,

show that $\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{x}{\sqrt{x-x^2}}\,dx = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (1+\sin\theta)\,d\theta$.

Hence, evaluate the integral.

11. (a) Show that $\int_0^1 x^2 e^x\,dx = e-2$.

(b) Prove that $\int_0^{\frac{\pi}{2}} x\cos x\,dx = \frac{\pi}{2}-1$.

12. Express $\frac{1}{1-x^2}$ in partial fractions. Hence, show that, if $-1 < x < 1$,

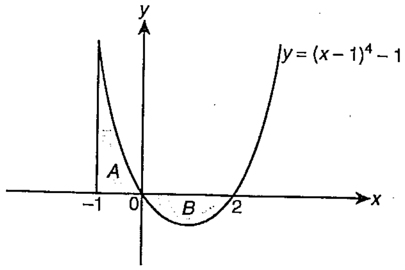
$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\ln(\frac{1+x}{1-x}) + c$, where c is the constant of integration

By integrating by parts, show that

$\int \frac{1}{1-x^2}\,dx = \frac{x}{1-x^2} - \int \frac{2x^2}{(1-x^2)^2}\,dx$.

Deduce the value of $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^2}\,dx$ correct to three significant figures.

13. The graph of $y = (x-1)^4 - 1$ is as shown below.

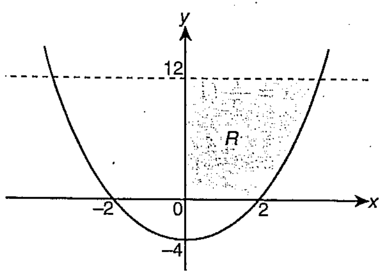


Find the total area of the shaded region A and B.

14. Sketch the graphs of the curves $y = (x-2)^2 + 1$ and $y = 6 - (x-3)^2$.
Find the coordinates of their points of intersection. Show that the area enclosed by the two arcs between their points of intersection is 9.

15. Find the area of the region in the first quadrant bounded by the curve $y = x^2 + 4$, the line $y = 8$ and the y-axis.
This region is rotated through 360° about the y-axis. Find the volume of revolution formed.

16. The graph shows the curve $y = x^2 - 4$. The region R is formed by the line $y = 12$, the x-axis, the y-axis and the curve $y = x^2 - 4$ for positive values of x.



The inside of a vase is formed by rotating region R through 360° about the y-axis. Each unit of x and y represents 2 cm.

(a) Write down an expression for the volume of revolution of region R about the y-axis.

(b) Find the capacity of the vase in litres

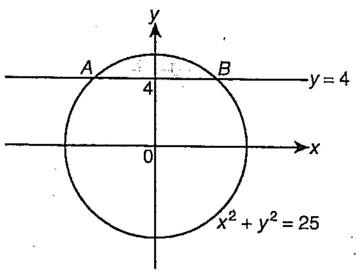
(c) Show that the vase is filled to $\frac{5}{6}$ of its internal height it is three-quarters full.

17. A mathematical model for a large garden pot is obtained by rotating through 360° about the y-axis the part of the curve $y = 0.1x^2$ which is between $x = 10$ and $x = 25$ and then adding a flat base. Units are in centimetres.

(a) Sketch the curve and shade the cross-section of the pot, indicating which line will form its base.

(b) Garden compost is sold in litres. Find the number of litres required to fill the pot to a depth of 45 cm. (Ignore the thickness of the pot).

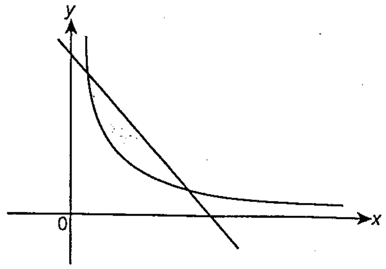
18. (a) Find the coordinates of A and B, the points of intersection of the circle $x^2 + y^2 = 25$ and the line $y = 4$.



(b) A napkin ring is formed by rotating the shaded area through 360° about the x-axis. Find the volume of the napkin ring.

19. The region bounded by the lines $x = 0$, $x = 1$, $y = 0$ and the curve $y = \frac{1}{2 - x}$ is denoted by R. Calculate the area of R and the volume of revolution formed when R is rotated through 360° about the x-axis.

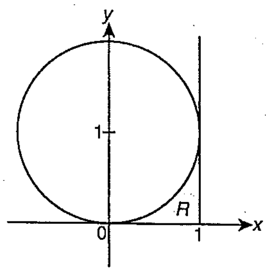
20.



The diagram shows a sketch of part of the curve $xy = 3$ and part of the line $y = 4 - x$. Use integration to find the area of the shaded region.

21. The equation $x^2 + y^2 = 1$ represents a circle with centre 0, the origin and radius 1 unit. By considering an appropriate region of the circle, show that,

$$\int_0^1 \sqrt{1 - x^2}\, dx = \frac{\pi}{4}.$$



The diagram shows a circle with equation $x^2 + (y - 1)^2 = 1$. The region R is bounded by the circle, the x-axis and the line $x = 1$. Show that the volume of the solid formed when R is rotated through 360° about the x-axis is given by

$$\pi \int_0^1 (2 - x^2 - 2\sqrt{1 - x^2})\, dx.$$

Hence, find this volume, giving your answer in terms of π.

22. Sketch the curve $y = 1 + 2e^{-x}$, showing clearly the behaviour of the curve as $x \to +\infty$. Find the area of the finite region enclosed by the curve and the lines $x = 0$, $x = 1$ and $y = 1$. Find the volume formed when this region is rotated completely about the line $y = 1$.

23. (a) Evaluate

(i) $\int_0^1 \frac{1 + x}{1 + 2x}\, dx$ (ii) $\pi \int_0^{\frac{\pi}{3}} \sin x \cos^2 x\, dx$

(b) Sketch the arc of the curve $y = 2x - x^2$ for which y is positive. Find the area of the region which lies between this arc and the x-axis. If this region is rotated completely about the x-axis, find the volume of the solid of revolution generated.

24. Obtain an approximate value of $\int_0^4 \frac{1}{1 + \sqrt{x}}\, dx$ by using the trapezium rule with 5 ordinates, giving your answer correct to three significant figures.

25. Use the trapezium rule with ordinates at $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$ and $\frac{\pi}{2}$ to estimate the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\sin \theta}\, d\theta$.

26. (a) Given that $I = \int_{-1}^1 \frac{1}{1 + e^{-x}}\, dx$, show that the estimate of I obtained by using the trapezium rule with 3 ordinates is 1.

(b) By means of the substitution $u = e^x$, show that the estimate obtained in (i) is correct.

27. Given that $f(x) = 2x^3 - 7x^2 + x + k$ and $(x - 2)$ is a factor of $f(x)$, find the value of k and factorise $f(x)$ completely.

Sketch the curve $y = f(x)$. (You are not required to find the coordinates of the stationary points)

Use the trapezium rule with 4 ordinates to find an approximation to $\int_{-1}^{2} f(x)\, dx$.

28. (a) If $I = \int_0^1 (x^2 + 1)^{-\frac{3}{2}}\, dx$, use the trapezium rule with 3 ordinates to estimate the value of I, giving your answer correct to two significant figures.

(b) By using the trapezium rule with the same ordinates as part (a), estimate the volume of solid formed to when the region bounded by the curve $y = (x^2 + 1)^{-\frac{3}{2}}$, the axes and the line $x = 1$ is rotated completely about the x-axis, giving your answer correct to two significant figures.

Revision Exercise

1. (a) $\frac{1}{2}\ln|2x-a|+c$ (b) $\frac{1}{3}\ln|3x^2+9x-1|+c$

   (c) $e^x-2e^{-2x}+c$ (d) $\frac{(2x-1)^2(x+1)}{3}$

   (e) $-\sqrt{4-x^2}$ (f) $-\frac{1}{3}e^{-x^3}$

   (g) $\ln\frac{x}{\sqrt{(2x+1)}}$ (h) $x^2-x+\ln|x+1|+c$

2. (a) $\frac{1}{2}\ln\frac{32}{27}$ (b) $\frac{4}{15}$

   (c) $\frac{1}{9}(1-4e^{-3})$ (d) $\frac{4}{15}(1+\sqrt{2})$

   (e) $\frac{1}{2}(e^2+3)$ (f) $\frac{1}{2}\ln 10$

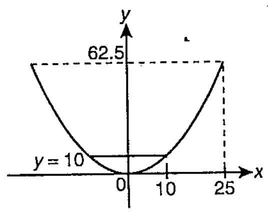
   (g) $2\frac{1}{4}$ (h) $\ln\frac{2}{3}$

3. $\frac{10}{3}$
5. $\frac{1}{3}$
6. $-\pi$, π
7. 2
8. $\frac{3}{5(x+1)}-\frac{1}{5(2x-3)}$, $\frac{3}{5}\ln 4-\frac{7}{10}\ln 3$
9. (a) $2\ln\frac{7}{3}$ (b) $4(\sqrt{7}-\sqrt{3})$

   (c) $2-\frac{3}{2}\ln\frac{7}{3}$
10. (b) $\frac{\pi}{6}$
12. 0.0587
13. 6.8
15. $\frac{16}{3}$, 8π
16. (a) $V=\pi\int_0^{12}x^2\,dy$ (b) 3 litres

    (c) $\int_0^{10}\pi(y+4)\,dy=90\pi$

    $=\frac{3}{4}(120\pi)$
17. (a)



    (b) $45-9l$

18. (a) $A(-3, 4)$, $B(3, 4)$ (b) 36π
19. $\ln 2$, $\frac{1}{2}\pi$
20. $4-3\ln 3$
21. $\frac{\pi}{6}(10-3\pi)$
22. $2-\frac{2}{e}$; $2\pi(1-\frac{1}{e^2})$
23. (a) (i) $\frac{1}{2}+\frac{1}{4}\ln 3$ (ii) $\frac{1}{3}$

    (b) $\frac{4}{3}$, $\frac{16}{15}\pi$
24. 1.95
25. 0.945
27. $k=10$; $f(x)=(x-2)(2x-5)(x+1)$; 18
28. (a) 0.70 (b) 1.7