Quick Review 1.4

1. Express the following expressions in their equivalent logarithmic form:
   (a) $2^5 = 32$
   (b) $5^3 = 125$
   (c) $32^{0.2} = 2$
   (d) $9^{\frac{3}{2}} = 27$

2. Express the following expressions in their equivalent exponential form:
   (a) $\log_4 256 = 4$
   (b) $\log_3 243 = 5$
   (c) $\log_{0.5} 0.125 = 3$
   (d) $\log_4 \left(\frac{1}{16}\right) = -2$

3. Without using calculators, evaluate the following:
   (a) $\log_3 243$
   (b) $\log_5 \frac{1}{125}$
   (c) $\log_{\frac{1}{2}} 64$
   (d) $\log_{27} 3$

4. Find the logarithms of:
   (a) $243\ \sqrt[5]{9}$ to base $3\sqrt{3}$
   (b) 64 to base $2\sqrt{2}$
   (c) 64 to base $\frac{1}{2\sqrt{2}}$
   (d) $\sqrt[3]{a^{\frac{15}{2}}}$ to base $\sqrt{a}$

5. Evaluate:
   (a) $5^{\,3\log_5 2}$
   (b) $a^{-3\log_a 3}$
   (c) $b^{-2\log_b \left(\frac{1}{5}\right)}$
   (d) $9.4^{\log_{9.4} \sin x}$

6. Express the following in terms of $\log x$, $\log y$ and $\log z$:
   (a) $\log (\sqrt{x}y^3/z)$
   (b) $3 \log (y^2z^5/x^3)^{\frac{1}{3}}$
   (c) $\log (\sqrt{x^2y^3z^4})$
   (d) $\log (x^2\sqrt{y}/z^3)$

7. Simplify the following expressions:
   (a) $\log (28) - \log (21) + \log 6$
   (b) $0.5 \log 25 + 3 \log 2 - 0.5 \log 16$
   (c) $2 \log\left(\frac{1}{3}\right) + 3 \log 3 + \frac{1}{2} \log 16$
   (d) $\dfrac{\log_3 729}{\log_3 27}$
   (e) $\dfrac{\log_4 32}{\log_4 2}$
   (f) $\frac{1}{2} \log (x-1) - \log (x^2 - 1)$
   (g) $\frac{1}{2} \log (x^2 + 2x + 1) - 2 \log (x - 1) - \log (x + 1)$
   (h) $\log \frac{5}{32} - 4 \log \frac{5}{4} + 3 \log \frac{5}{4} - 4 \log \frac{9}{4}$

8. Without using a calculator evaluate each of the following:
   (a) $\log_2 30 + 2 \log_{10} \frac{5}{16} - 3 \log_2 \frac{25}{32} + \log_2 \frac{125}{96}$
   (b) $\log_2 3 \times \log_3 4$
   (c) $\log_2 125 \times \log_5 32$
   (d) $(\log_3 5 + \log_3 \sqrt{5})(\log_{\sqrt{5}} 3)$
   (e) $(\log_3 5 + \log_9 25)(\log_5 9 + \log_{25} 3)$

9. Prove that
   $$\log \frac{x^2}{yz} - \log \frac{zx}{y^2} + \log \frac{z^2}{xy} = 0$$

10. Given that $\log_3 2 = x$ and $\log_3 5 = y$, find in terms of x and y:
    (a) $\log_3 6.4$
    (b) $\log_{100} 2$