Exercise 7.13

1. Find the minimum value of $x^2 + 2y^2$ if x and y are related by $x + 2y = 1$.

2. If $xy = 4$, find the maximum and minimum values of $P = x + y$ where x and y are positive.

3. If $V = x^2 h$ and $h + x = 6$, find the maximum and minimum values of V.

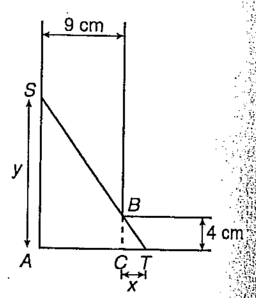
4. A window consists of a rectangle of width $2x$ and height a, surmounted by a semicircle of diameter $2x$. If the perimeter of the window is P, show that the area of the window is given by $A = Px - 2x^2 - \frac{1}{2}\pi x^2$.
   If x varies and P is a constant, find, in terms of P, the greatest possible area of the window.

5. A solid circular cylinder has a given volume. Show that its total surface area will be least when its height is equal to the diameter of the base.

6. The lengths of the sides of a rectangular sheet of metal are 8 cm and 3 cm. A square of side x cm is cut from each corner of the sheet and the remaining piece is folded to make an open box.
   (a) Show that the volume V of the box is given by $V = 4x^3 - 22x^2 + 24x$.
   (b) Find the value of x for which the volume of the box is a maximum. Calculate the maximum volume.

7. An open rectangular box is made of very thin sheet metal. Its volume is 128 cm^3, its width is x cm and its length of $4x$ cm. Obtain an expression for its depth in terms of x. Show that the total surface area of its base, its ends and its sides is equal to $(4x^2 + \frac{320}{x})$ cm^2.
   Calculate the dimensions of the box for which the surface area is a minimum.

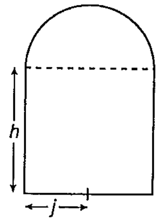
8. The diagram on the right shows the junction of two roads which are perpendicular to each other. The width of the roads are 9 m and 4 m respectively. S and T are two variable points such that SBT is a straight line. If $CT = x$ m and $AS = y$ m, express y in terms of x and find the value of x such that $TA + AS$ is a minimum.



9. The diagram on the right shows the cross-section of a cylindrical solid with radius r and height h. A hemisphere with radius r is attached to the upper surface of the cylinder. If the volume of the combined solid is V, show that the total surface area of the combined solid is

$$\frac{2V}{r} + \frac{5\pi r^2}{3}.$$

   If r varies and V is a constant, prove that the surface area is minimum when $r = h$.



10. A rectangular garden consists of a rectangular lawn of area 72 m^2 surrounded by a concrete path. The path is 2 m wide at two opposite edges of the garden and 1 m wide along each of the other two edges. Find the dimensions of the garden of smallest area satisfying these requirements.

Exercise 7.13

1. $\frac{1}{3}$
2. 4 minimum
3. 0 minimum, 32 maximum
4. $\frac{P^2}{2(4 + \pi)}$
5. $(200\pi)^{-\frac{1}{2}}$
6. (b) $x = \frac{2}{3}$, $V_{max} = 7\frac{11}{27}$ cm^3
7. $32x^{-2}$, width 3.42 cm, length 13.68 cm, depth 2.74 cm
8. $y = \frac{4x + 36}{x}$; 6
10. 6 m × 12 m