Exercise 5.7

1. Sketch the curve represented by each of the following equations.
   (a) $y^2 = 8x$ (b) $y^2 + 12x = 0$
   (c) $2y^2 = 9x$ (d) $3y^2 + 8x = 0$

2. Find the vertex of each of the following parabolas and sketch its curve.
   (a) $y^2 = 4(x - 3)$ (b) $(y + 2)^2 = 2(x - 1)$
   (c) $y^2 - 2y = 8x + 7$ (d) $y^2 = 6y + x - 5$

3. Find, in Cartesian form, of each of the following parametric equations.
   (a) $x = 2t^2, y = 4t$ (b) $x = \frac{3}{2}t^2, y = 3t$
   (c) $x = 3p^2, y = 6p$ (d) $x = \frac{5}{3}t^2, y = \frac{10}{3}t$

4. Obtain the parametric equations for each of the following curves.
   (a) $y^2 = 6x$ (b) $y^2 = 9x$
   (c) $2(y - 1)^2 = 15x$ (d) $3y^2 = 8(x - 1)$
   (e) $(y - 1)^2 = 16(x - 2)$

5. Find the equations of the tangents and normals for each of the following parabolas at the given points.
   (a) $y^2 = 8x$; $(2, 4)$ (b) $y^2 = 12x$; $(3, 6)$
   (c) $y^2 = 10x$; $(\frac{5}{2}, -5)$ (d) $y^2 = 6ax$; $(\frac{3}{2}a, 3a)$

6. The points $(1, 6)$ and $(81, -54)$ lie on the parabola $y^2 = 36x$. Find the equation of the tangents to the parabola at these points, and hence show that the tangents are perpendicular to each other. Find the point of intersection of the tangents.

7. Find the equations of the tangent and normal to the parabola $y^2 = 4x$ at the point $(t^2, 2t)$. If P is the point with $t = \sqrt{2}$, find the coordinates of the point Q where the normal at the point P meets the parabola again. If O is the origin, show that OP is perpendicular to OQ.

8. If PQ is a chord which is a normal at the point $(a, 2a)$, which lies on the parabola $y^2 = 4ax$, find the length of PQ. Find also the area of the triangle formed by the chord and the tangents at P and Q.

9. The line $y - 2x + 4a = 0$ intersects the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ and the point $Q(aq^2, 2aq)$. Find the value of $p + q$ and pq. Hence, find the coordinates of the mid-point of PQ.

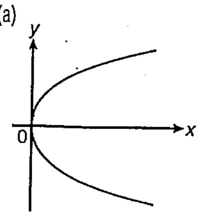
10. $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are two points on the parabola $y^2 = 4ax$ and the line PQ passes through the point $(a, 0)$. Show that if the tangents at P and Q meet at T, the locus of T is a straight line and find its equation.

11. If the normal at $P(ap^2, 2ap)$ to the parabola $y^2 = 4ax$ meets the curve again at $Q(aq^2, 2aq)$, prove that $p^2 + pq + 2 = 0$. Prove that the equation of the locus of the point of intersection of the tangents at P and Q to the parabola is $y^2(x + 2a) + 4a^3 = 0$.

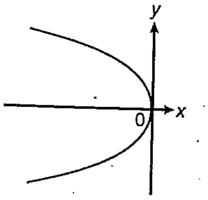
12. The line $y = mx + c$ intersects the parabola $y^2 = 4ax$ at the points P and Q. Show that the coordinates of the mid-point of PQ is $(\frac{2a - mc}{m^2}, \frac{2a}{m})$.
    If this mid-point is M, find the locus of M when m varies and $c = 1$.

Exercise 5.7

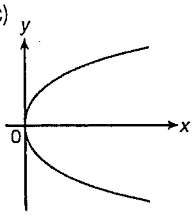
1. (a)



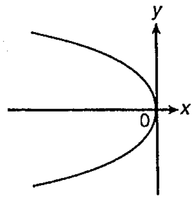
(b)



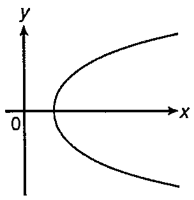
(c)



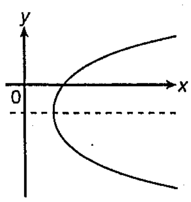
(d)



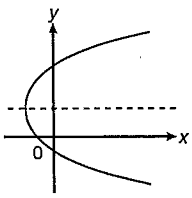
2. (a) (3, 0)



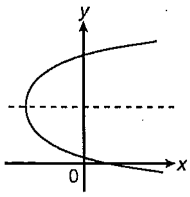
(b) (1, −2)



(c) (−1, 1)



(d) (−4, 3)



3. (a) $y^2 = 8x$ (b) $y^2 = 6x$
(c) $y^2 = 12x$ (d) $3y^2 = 20x$

4. (a) $x = \frac{3}{2}t^2,\ y = 3t$
(b) $x = \frac{9}{4}t^2,\ y = \frac{9}{2}t$
(c) $x = \frac{15}{8}t^2,\ y = \frac{15}{4}t + 1$
(d) $x = \frac{2}{3}t^2 + 1,\ y = \frac{4}{3}t$
(e) $x = 2(2t^2 + 1),\ y = 8t + 1$

5. (a) $y = x + 2;\ y + x = 6$
(b) $y = x + 3;\ y + x = 9$
(c) $2y + 2x + 5 = 0;\ 2y = 2x - 15$
(d) $2y = 2x + 3a;\ 2y + 2x = 9a$

6. $y = 3x + 3,\ 3y + x + 81 = 0,\ (-9, -24)$

7. $ty = x + t^2,\ tx + y = t^3 + 2t$
$(8, -4\sqrt{2})$

8. $8\sqrt{2}\,a,\ 32a^2$

9. $p + q = 1,\ pq = -2$
$(\frac{5}{2}a,\ a)$

10. $x = -a$

12. $2ax = y^2 - y$