Exercise 7.15

1. The side of a cube is increasing at a constant rate of 0.1 m s^{-1}. Find the rate of increase of its volume when each side is 2 m long.

2. The volume of a spherical balloon is increasing at a constant rate of $0.05 \text{ m}^3 \text{ s}^{-1}$. Find the rate of increase of its radius when the volume of the balloon is 0.008 m^3.

3. A spherical balloon is being inflated in such a way that its volume is increasing at a constant rate of $8 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the surface area of the balloon when its radius is 10 cm.

4. The radius of a circular oil slick is increasing at 1.5 m s^{-1}. Taking π to be 3.14, find, to 2 significant figures, the rate at which the area of the oil slick is increasing when its radius is 300 m.

5. An inverted right circular cone of semi-vertical angle 45° is collecting water from a tap at a steady rate of $18\pi \text{ cm}^3 \text{ s}^{-1}$. Find the rate at which the depth h of the water is rising when $h = 3$ cm.

6. A hemispherical bowl of radius 6 cm contains water which is flowing into it at a constant rate. When the height of the water is h cm, the volume V of the water in the bowl is given by

$$V = \pi\left(6h^2 - \frac{1}{3}h^3\right) \text{ cm}^3.$$

Find the rate at which the water level is rising when $h = 3$, given that the time taken to fill the bowl is 1 minute.

7. A ladder 5 m long is leaning against a vertical wall. The bottom of the ladder is pulled along the ground away from the wall at a constant rate of 0.4 m s^{-1}. How fast will the top of the ladder be falling at the instant when its bottom is 3 m away from the wall?

8. A flask in the shape of a cone of height 20 cm and radius 8 cm is held vertex downwards. Show that when the depth of water in the flask is x cm, the volume of water is $\frac{4}{75}\pi x^3 \text{ cm}^3$. Water leaks out from the vertex at the rate of $2 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of change of the depth of the water when the depth is 10 cm.

9. A hemispherical bowl of radius 12 cm is initially full of water. Water runs out of a small hole at the bottom of the bowl at a rate of $48\pi \text{ cm}^3 \text{ s}^{-1}$. When the depth of the water is x cm, show that the depth is decreasing at a rate of $\frac{48}{x(24 - x)} \text{ cm s}^{-1}$.
Find the rate at which the depth is decreasing when
(a) the bowl is full,
(b) the depth is 6 cm.
[When the depth of water in the hemispherical bowl with radius r is x cm, the volume of the water is $\frac{1}{3}\pi x^2(3r - x) \text{ cm}^3$]

10. Two variables p and q are connected by the relation $\frac{1}{p} + \frac{1}{q} = \frac{1}{k}$ where k is a constant. Given that $k = 10$ and p decreases at the rate of 2 cm s^{-1}, find the rate of change of q when $p = 40$.

Exercise 7.15

1. $1.2 \text{ m}^3 \text{ s}^{-1}$
2. 0.26 m s^{-1}
3. $1.6 \text{ cm}^2 \text{ s}^{-1}$
4. $2800 \text{ m}^2 \text{ s}^{-1}$
5. 2 cm s^{-1}
6. $\frac{4}{45} \text{ cm s}^{-1}$
7. 0.3 ms^{-1}
8. 0.04 cm s^{-1}
9. (a) $\frac{1}{3} \text{ cm s}^{-1}$ (b) $\frac{4}{9} \text{ cm s}^{-1}$
10. $\frac{2}{9} \text{ cm s}^{-1}$