Exercise 8.10

1. Find the volume of the solid of revolution formed when the region defined in terms of the lines which form its boundaries is rotated though 360° about the x-axis.
   (a) $y = 2x$, the x-axis and the lines $x = 1$ and $x = 3$
   (b) $y = x^2 + 1$, the x-axis and the lines $x = -1$ and $x = 1$
   (c) $y = \sqrt{x}$, the x-axis and the line $x = 4$

2. Find the volume of the solid of revolution formed when the region bounded by the curves and the given lines is rotated completely about the y-axis.
   (a) $y = 3x$, the y-axis and the lines $y = 3$ and $y = 6$
   (b) $y = x - 3$, the y-axis, the x-axis and the line $y = 6$
   (c) $y = x^2 - 2$, the y-axis and the line $y = 4$.

3. The region bounded by the two curves is rotated completely about the x-axis. Find the volume of the solid formed.
   (a) $y = x(6 - x)$ and $y = 3x$
   (b) $y^2 = 4x$ and $y = 2x$
   (c) $y^2 = 4x$ and $x^2 = 4y$.

4. The region R in the first quadrant is bounded by the y-axis, the x-axis, the line $x = 3$ and the curve $y^2 = 4 - x$. Calculate the volume formed when R is rotated about the y-axis through one revolution.

5. A hemispherical bowl is formed by rotating the bottom half of the circle $x^2 + y^2 = 100$ about the y-axis.
   (a) Find the volume of the bowl.
   (b) The bowl is filled with water to a depth of 8 cm. Find the volume of water in the bowl.

6. Sketch on the same axes, show that part of the curve $y = 16 - x^2$ and the line $y = 6x$ lies in the first quadrant. Shade the area. The region bounded by the curve and the line is rotated completely about the x-axis. Find the volume generated, leaving your answer as a multiple of π.

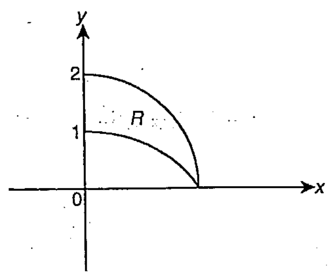
7. The area bounded by the curve $y = \tan x$, the x-axis and the ordinate $x = \frac{\pi}{3}$ is rotated about the x-axis. Calculate the volume of the solid formed.

8. Calculate the volume generated when the finite region enclosed by the curve $y = 1 + 2e^{-x}$ and the lines $x = 0$, $x = 1$ and $y = 1$ is revolved completely about the x-axis.

9. Sketch the curve $y = e^x$ and $y = e^{-x}$ for $-2 \leqslant x \leqslant 2$. The interior of a wine glass is formed by rotating the curve $y = e^x$ from $x = 0$ to $x = 2$ about the y-axis. If the units are in centimetres, find, correct to 2 significant figures, the volume of liquid that the glass contains when full.

10. Sketch the curve whose equation is
$$y = 1 - \frac{1}{x+2},$$
indicating any asymptotes which the curve possesses.
The region bounded by the curve, the x-axis and the ordinates $x = 0$ and $x = 2$ is denoted by R.
Find the volume swept out when R is rotated about the x-axis through an angle of 2π.

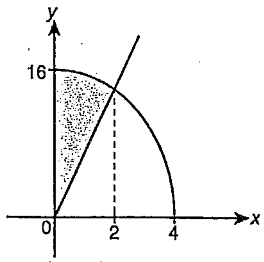
11. The diagram shows the region R in the first quadrant bounded by the curves $y = \frac{1}{4}(4 - x^2)$, $y = \frac{1}{2}(4 - x^2)$ and the y-axis. Calculate the volume of the solid formed when R is rotated through an angle of 2π about the y-axis.



12. A chord of the circle $x^2 + y^2 = r^2$ is parallel to the x-axis and of the length $2l$. The minor segment cut off by this chord is rotated about the x-axis to form a solid of revolution. Prove that its volume is $\frac{4}{3}\pi l^3$.

Exercise 8.10

1. (a) $\frac{104}{3}\pi$ units^3 (b) $\frac{56}{3}\pi$ units^3
   (c) 8π units^3
2. (a) 7π units^3 (b) 234π units^3
   (c) 18π units^3
3. (a) $\frac{243}{5}\pi$ units^3 (b) $\frac{2}{3}\pi$ units^3
   (c) $\frac{96}{5}\pi$ units^3
4. $\frac{188}{15}\pi$ units^3
5. (a) $\frac{2000}{3}\pi$ units^3 (b) $\frac{1408}{3}\pi$ units^3
6.



$337\frac{1}{15}\pi$ units^3

7. 2.15 units^3
8. $\pi(6 - 4e^{-1} - 2e^{-2})$
9. 40
10. $\pi(\frac{9}{4} - 2\ln 2)$
11. 2π