Topic 17: Exercises on Volumes and Shells
Level 3, Part 2

1. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region enclosed by the circle $(x-b)^2 + y^2 = a^2$ (where $b > a$) about the y- axis.

$2\pi^2 a^2 b$ units^3

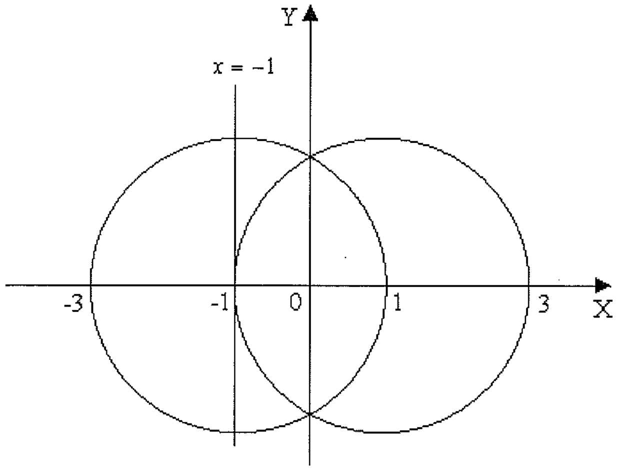
2. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : (x-1)^2 + \frac{y^2}{4} \leq 1\}$ about the y- axis.

$4\pi^2$ units^3

3. $f(x)$ is an even function. The area founded by $y = f(x)$ and the x- axis is rotated about $x = -a$. The strips of width δx form cylindrical shells of the same height . Show that the volume of the solid is given by $V = 4\pi a \int_0^a f(x)dx.$

4. The centres of two circles , each of radius 2 cm , are 2 cm apart. The region common to the two circles is rotated about one of the tangents to this region which is perpendicular to the line joining the centres. Show that the volume of the solid formed is given by

$V = 8\pi \int_0^1 \sqrt{4-(x+1)^2}\, dx$, and hence find the volume.



Hint: Use the formula obtained in Question 3.

$\frac{4\pi}{3}\left(4\pi - 3\sqrt{3}\right)$ cubic cm.

5. The region between the two curves $y=\frac{1}{2}\left(e^x+e^{-x}\right)$ and $y=\frac{1}{2}\left(e^x-e^{-x}\right)$ bounded by the y-axis and the line $x=1$ is rotated about the y- axis. Use cylindrical shells to show that the volume of the solid generated is given by $V=2\pi\int_0^1 xe^{-x}\,dx$, and hence calculate this volume.

$2\pi\left(1-2e^{-1}\right)$ units^3

8. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region
$\{(x, y) : 0 \le x \le 1, 0 \le y \le x - x^2\}$ about the x-axis.

$\frac{\pi}{30}$ units^3