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Undersea Defence Technology
13-15 May 2019
Stockholmsmässan, Sweden

Prediction and Representation of Array Performance under Sensor Failure

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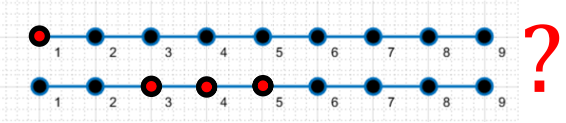
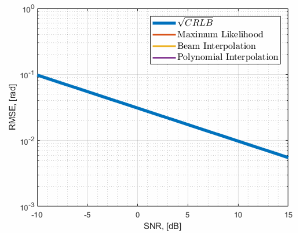
Outline

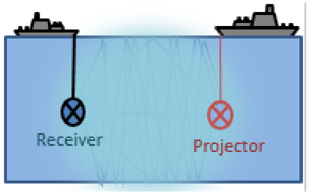
- **Problem definition**

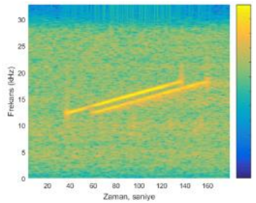


- **Performance bounds**
  - Bayesian bounds
  - Deterministic bounds

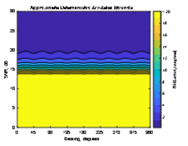


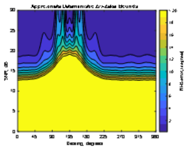
- **Signal models**
  - Fading channels





- **Results and conclusion**





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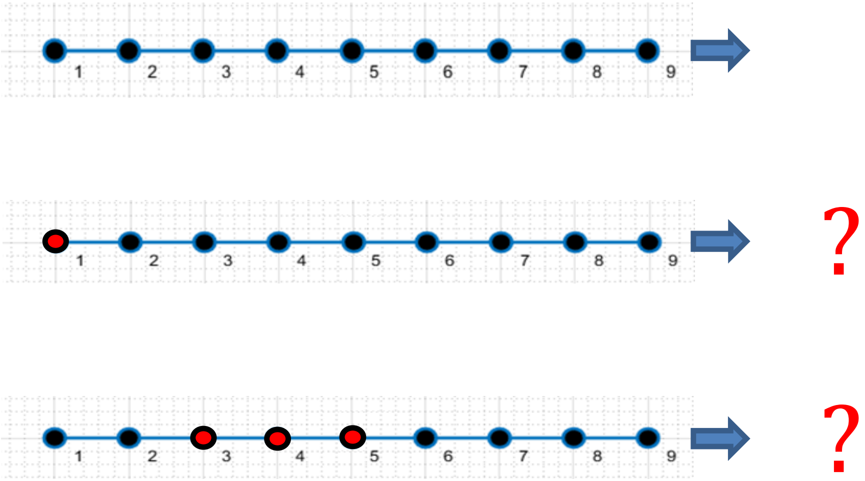
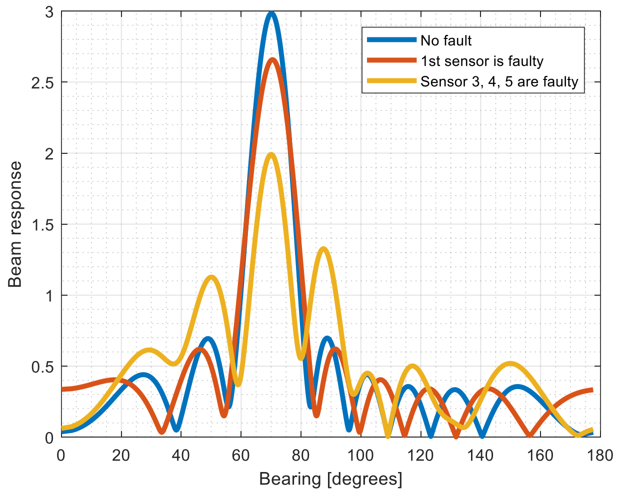
A practical problem

- How much should the sonar operator trust the system when some of the sensors fail (or disabled)?





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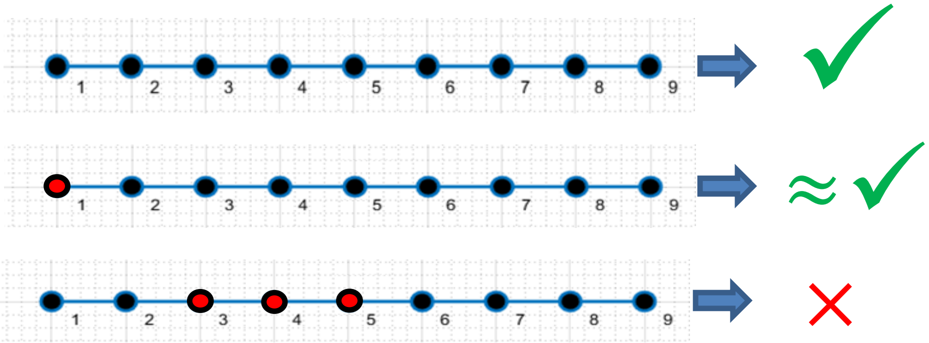
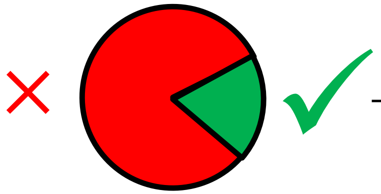
How to report?

- In general the user manual of a given system provides the following information;
  - The system is operational, up to N sensor failures.
  - The system is partially operational, up to M sensor failures.
  - The system is not operational, after K sensor failures.



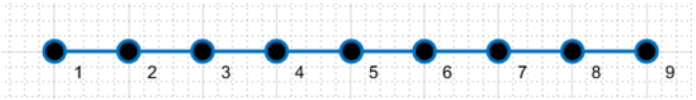
How to report?

- The drawback here is these bounds are loose.
  - i.e. The system may tolerate the loss of sensors in certain angular sector.



$\rightarrow$ *Still usable in this sector*

- The bounds have to be loose, as the number of possible combinations become very large, as number of sensors increase.



$\Rightarrow$ 2^N subsets

Performance prediction

- System performance can be assessed using performance lower bounds.
  - Two classes of performance bounds exist
    - **Bayesian bounds**
      - The parameter to be estimated is a random variable with a known a-priori distribution.
    - Deterministic bounds
      - The parameter to be estimated is a non-random parameter

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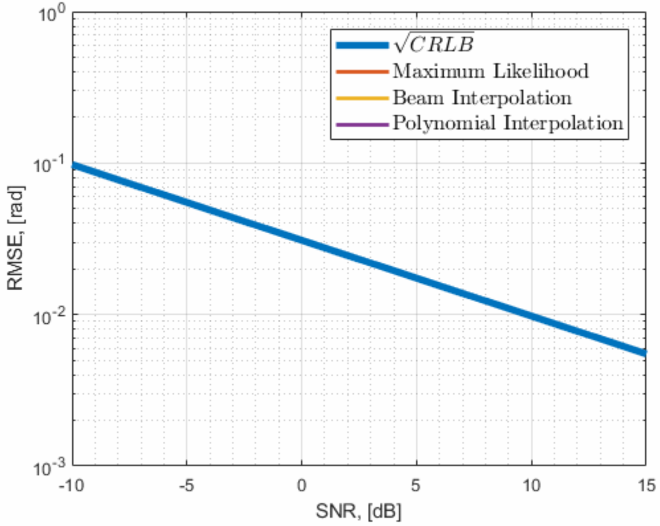
Performance Prediction

- Deterministic CRLB can be evaluated.
  - Bound is optimistic,
    - Under low SNR.
    - **Under fading conditions.**

- Deterministic version of the Ziv-Zakai Bounds can be used.
  - **Tighter lower bound, even at low SNR conditions!**



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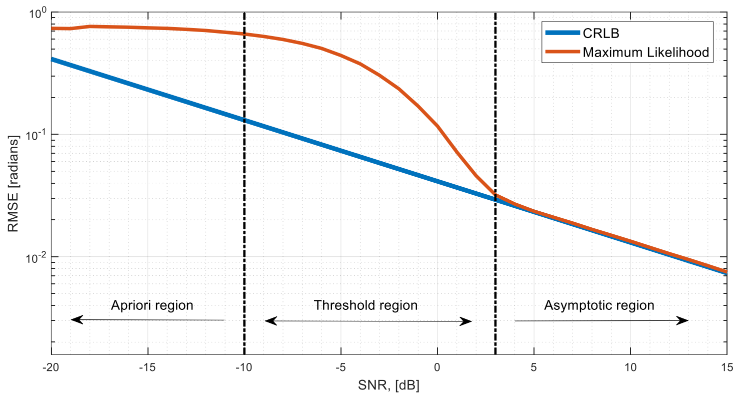
SNR Regions



- CRLB can not model gross-error events, since it only considers the second derivative of the beampattern at the mainlobe.
  - Consequently the errors produced by side-lobe jumps (gross errors at low SNR values) can not be modeled.
    - CRLB is not a tight bound at low SNR values.

- Performance bounds are generally divided into three regions w.r.t. SNR:

Apriori region: Region in which the estimate is uniformly distributed in the a priori domain of the unknown parameter (region of low SNRs).

Threshold region: Region of transition between the apriori and asymptotic regions (region of medium SNRs). The mean squared error is dominated by gross error events.

Asymptotic region: Region in which the CRLB is achieved (region of high SNRs). Gross error probability is negligibly small.

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Signal model

The signal model under Rician fading is as follows;

$$\tilde{y}_{nk} = \tilde{s}_k \boldsymbol{\eta_n} e^{j\theta_c} + \widetilde{\boldsymbol{N}}_{\boldsymbol{nk}}$$

$$\eta_n \sim CN(\bar{\eta}_{I_n} + j\bar{\eta}_{Q_n}, 2\sigma^2)$$

$$\tilde{y}_{nk} = \underbrace{\tilde{s}_k(\bar{\eta}_{I_n} + j\bar{\eta}_{Q_n})e^{j\theta_c}}_{\text{Specular component}} + \underbrace{\tilde{s}_k(\bar{\xi}_{I_n} + j\bar{\xi}_{Q_n})e^{j\theta_c}}_{\text{Random component}} + \tilde{N}_r$$

$$K = \frac{specular\ power}{random\ power} = \frac{(\bar{\eta}_I)^2 + (\bar{\eta}_Q)^2}{var(\eta_I) + var(\eta_Q)} = \frac{m_I^2 + m_Q^2}{2\sigma^2}$$

Rician factor → K

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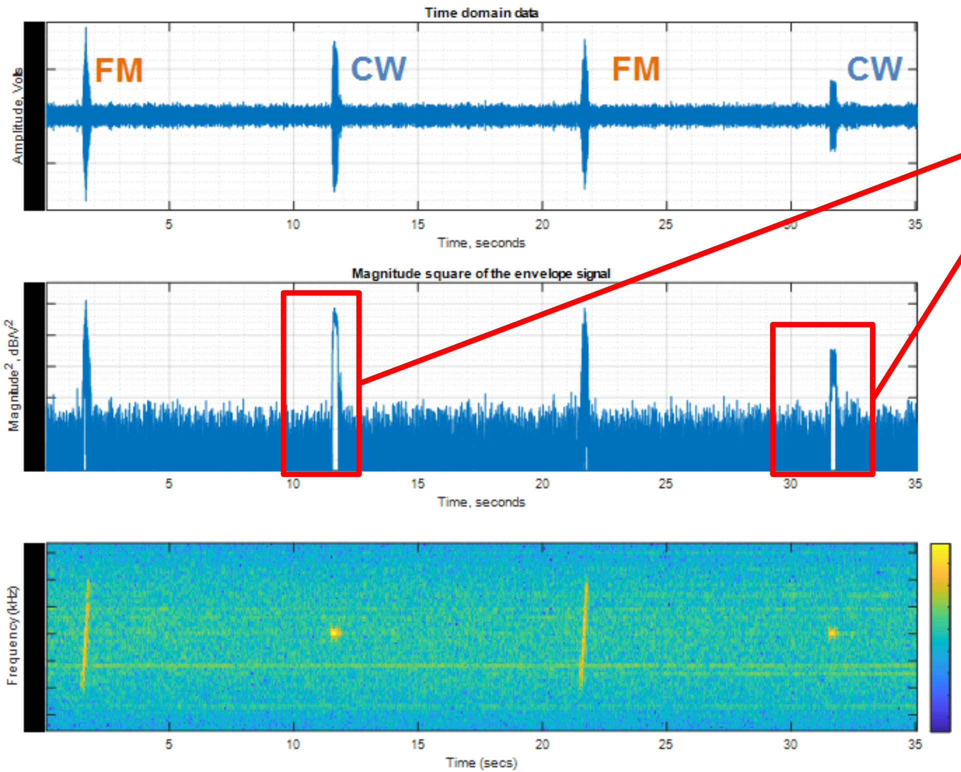
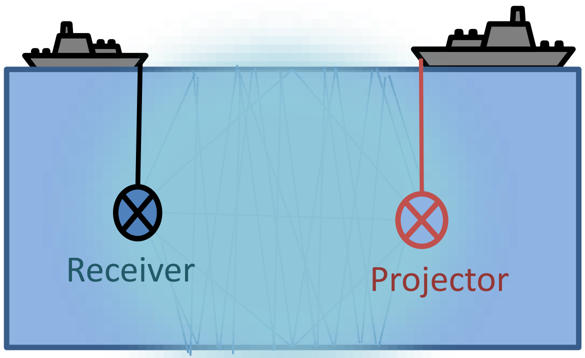
Channel Fading



Near 9 dB loss between two consecutive pulses.



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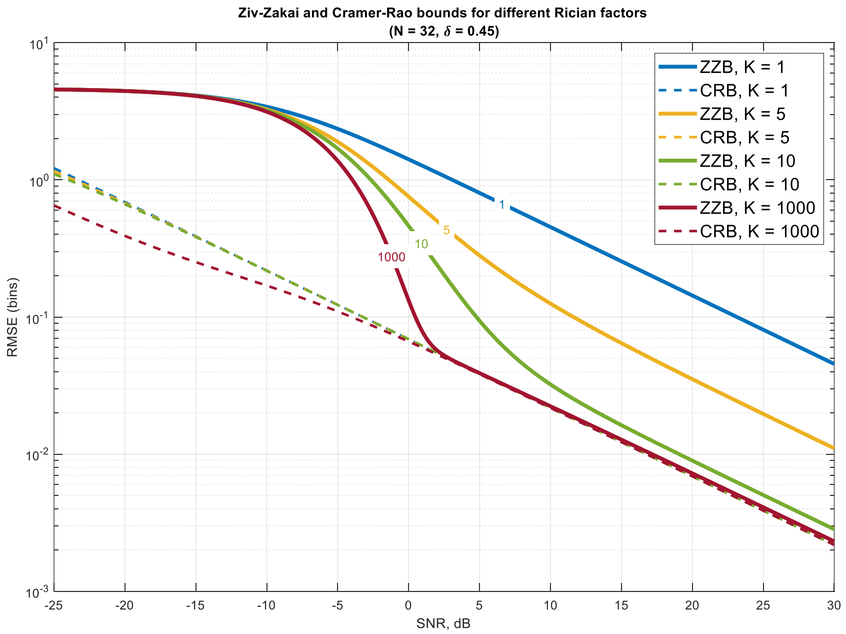
Ziv-Zakai and Cramer-Rao Bounds



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Cramer-Rao Bound under Rician Fading

The Fisher Information Matrix for a complex Gaussian pdf is as follows:

$$r_n = \eta_n A_s \exp(j\theta n) + n_n \qquad \boldsymbol{r} = \begin{bmatrix} r_0 \\ r_1 \\ \vdots \\ r_{N-1} \end{bmatrix}, \qquad \boldsymbol{a}_\theta = \begin{bmatrix} 1 \\ e^{j\theta} \\ \vdots \end{bmatrix}$$

$$\boldsymbol{\eta}_n \sim CN(\mu, 2\sigma_\eta^2), \qquad N, A_s \in \mathbb{R}$$

$$\boldsymbol{\mu} = E\{\boldsymbol{r}\} = \ldots$$

$$\mathbf{C}_x = E\{(\boldsymbol{r} \ldots \boldsymbol{\mu})^H\} = 2\sigma_\eta^2 A_s^2 \boldsymbol{a}_\theta \boldsymbol{a}_\theta^H + 2\sigma_n^2 \boldsymbol{I}_N$$

In summary, we can calculate this !

$$\text{FIM} = \left[\mathbf{I}_\xi\right]_{ij} = tr\left\{\mathbf{C}_x^{-1}(\xi)\frac{\partial \mathbf{C}_x(\xi)}{\partial \xi_i}\mathbf{C}_x^{-1}(\xi)\frac{\partial \mathbf{C}_x(\xi)}{\partial \xi_j}\right\} + 2\text{Re}\left\{\frac{\partial \boldsymbol{\mu}^H(\xi)}{\partial \xi_i}\mathbf{C}_x^{-1}(\xi)\frac{\partial \boldsymbol{\mu}(\xi)}{\partial \xi_j}\right\}$$

$$\boxed{\text{CRLB} = \text{FIM}^{-1}}$$

Ziv-Zakai Bounds

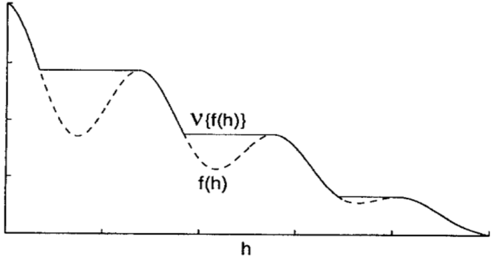
$$ZZB = \int_0^\infty v\{A(h)P_e(h)\}hdh$$

$$A(h) = \int_{-\infty}^\infty \min\big(p_u(u), p_u(u+h)\big)du$$

$$u{\sim}unif(0,2\pi) \rightarrow A(h) = \frac{2\pi - h}{2\pi}$$

$$\boxed{ZZB = \int_0^{2\pi} v\left\{P_e(h)\frac{2\pi - h}{2\pi}\right\}hdh}$$

*Valley filling function



$P_e(h)$: Minimum probability of error between deciding $H_0: u$ and $H_1: u+h$.

No angular dependence !

Dividing the parameter space into smaller intervals does not solve the issue, as the bound ignores gross errors larger than the sub-interval size.

*Figure taken from: **K.Bell, Y.Eprahim, H.L.Van Trees, "Explicit Ziv Zakai Lower Bound for Bearing Estimation", IEEE Transactions on Signal Processing, Vol. 44, No:11, Nov. 1996.**

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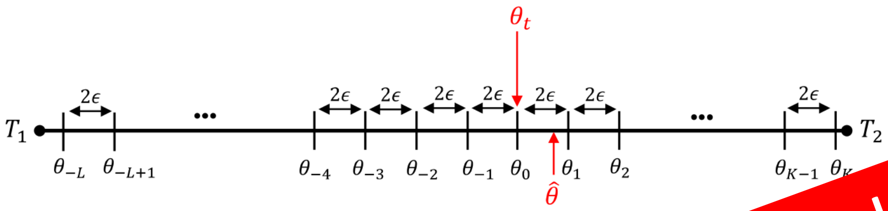
Approximate Deterministic ZZB



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...pace

In summary, we can calculate this !

$$\mathrm{MSE}_{\theta_t} = 2\int_0^{T_2-T_1} \epsilon P\{|\theta \ldots$$

$$\mathrm{MSE}_{\theta_t} \ldots \epsilon(P_{\mathrm{ML}}^{\text{e-point}}(\theta_t,\theta_t+2\epsilon)+P_{\mathrm{ML}}^{\text{e-point}}(\theta_t,\theta_t-2\epsilon))d\epsilon$$

$$= 2\int_0^{(T_2-T_1)/2} \epsilon(P_{\mathrm{ML}}^{\text{e-point}}(\theta_t,\theta_t+2\epsilon)+P_{\mathrm{ML}}^{\text{e-point}}(\theta_t,\theta_t-2\epsilon))d\epsilon$$

$$\geq 2\int_0^{(T_2-\theta_t)/2} \epsilon P_{\mathrm{ML}}^{\text{e-point}}(\theta_t,\theta_t+2\epsilon)d\epsilon + 2\int_0^{(\theta_t-T_1)/2} \epsilon P_{\mathrm{ML}}^{\text{e-point}}(\theta_t,\theta_t-2\epsilon)d\epsilon = \mathrm{ZZB_d}$$

This is an approximate lower bound for ML type estimators.

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Stein's unified analysis of error probability

Summary of the results for FSK in Stein's paper*:

$z_{1f} = b_t^f$ and $z_{2f} = b_l^f$

$$\bar{z}_{if} = m_{if} + j\mu_{if} = |\bar{z}_{if}|e^{j\theta_{if}}, \quad i = 1,2$$

$$S_{if} = \frac{1}{2}|\bar{z}_{if}|^2 = \frac{1}{2}(m_{if}^2 + \mu_{if}^2), \quad N_{if} = \frac{1}{2}\overline{|z_{if} - \bar{z}_{if}|^2}$$

$$\rho_f\sqrt{N_{1f}N_{2f}} = \frac{1}{2}\overline{(z_{1f} - \bar{z}_{1f})^*(z_{2f} - \bar{z}_{2f})}$$

$$\rho_f = \rho_{cf} + j\rho_{sf} = \frac{1}{2\sqrt{N_{1f}N_{2f}}}\overline{(z_{1f} - \bar{z}_{1f})^*(z_{2f} - \bar{z}_{2f})}$$

$$\frac{1}{2}\overline{(z_{1f} - \bar{z}_{1f})(z_{2f} - \bar{z}_{2f})} \ldots$$

ϕ …

$$\left\{\begin{matrix} a \\ b \end{matrix}\right\} = \frac{1}{2}\left[\frac{S_{1f} + S_{2f} + 2\sqrt{S_{1f}S_{2f}}\cos(\theta_{1f} - \theta_{2f} \ldots + \phi)}{N_{1f} + N_{2f} + 2 \ldots \sqrt{N_{1f}N_{2f}|\rho_f|^2}} \ldots N_{2f}|\rho_f|^2\right]$$

$$A = \frac{N_{1f} - N_{2f}}{\sqrt{(N_{1f} + N_{2f})^2 - 4N_{1f}N_{2f}|\rho_f|^2}}$$

In summary, we can calculate this !

$$P = \frac{1}{2}\left[1 - Q_1(\sqrt{b}, \sqrt{a}) + Q_1(\sqrt{b}, \sqrt{a})\right] - \frac{A}{2}\exp\left(-\frac{a+b}{2}\right)I_0(\sqrt{ab}).$$

Where Q_1 is the Marcum-Q function and I_0 is the modified Bessel function of the first kind.

*S. Stein, "**Unified analysis of certain coherent and non-coherent binary communication systems**," IEEE Trans. Inf. Theory, vol. IT-10, January 1964, pp. 43–51.

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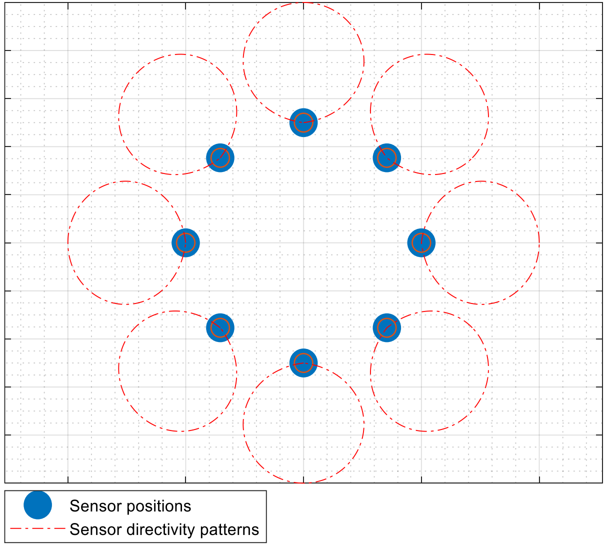
Online Performance Prediction



- Case study:

Circular array with directional sensors

Sensor patterns:

$$B_n(\phi) = \left|\frac{1}{2}\cos(\phi - \phi_n) + \frac{1}{2}\right|^2$$

$$\phi_n = \frac{(n-1)\pi}{6}, \quad n = 1, 2, \ldots, 12.$$

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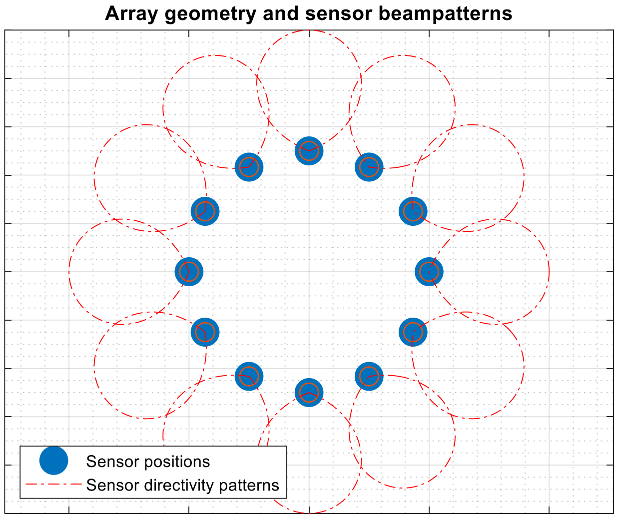

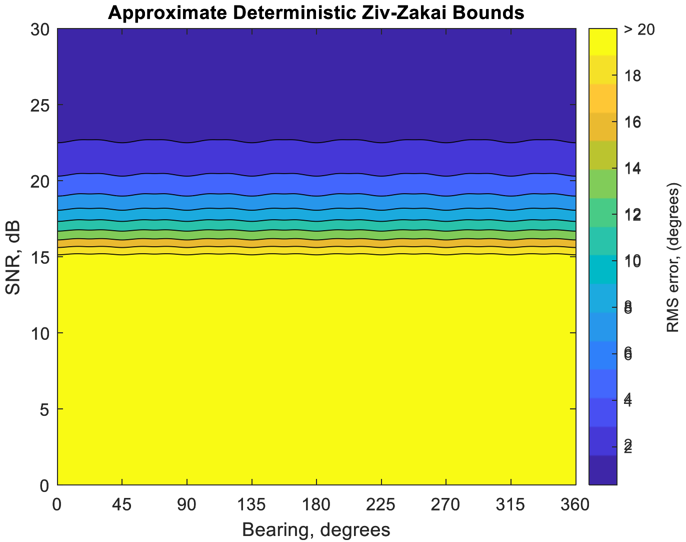

$K = 10$

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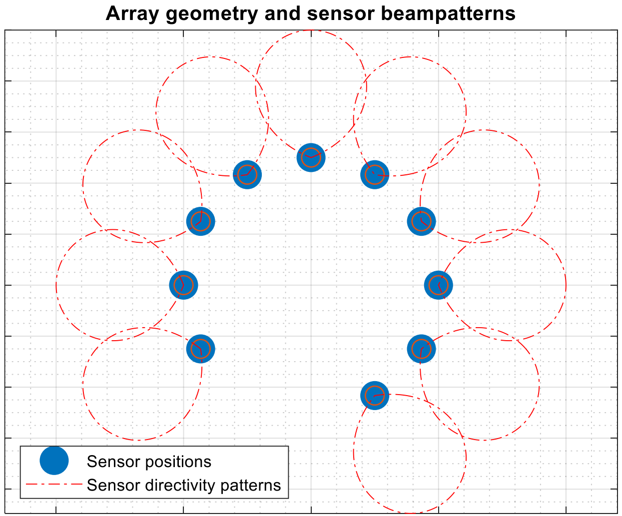
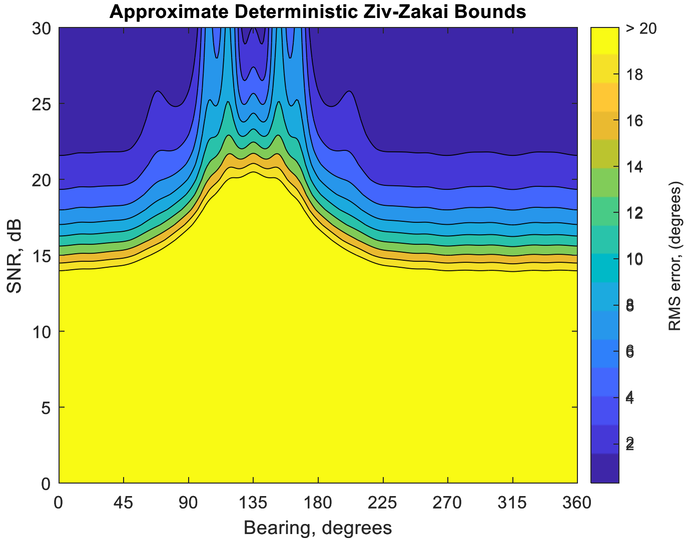
Online Performance Prediction





$K = 10$

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Summary

- Deterministic lower bounds can be utilized for on-line performance estimation of the system.
    - Extension of Ziv-Zakai bounds for an approximate deterministic bound is used in this work.
    - Closed form expressions are available.
        - Actual systems can be equipped with a system performance prediction tab, to provide the user with current system capabilities.