Enhancing Sonar resolution through smart signal processing

Abstract — Results from Compressive Sensing processing on measurement data from the fresh water lake Vättern (Sweden) are shown. A brief introduction to the methods used are given in terms of modelling, sparsity and optimization. Two different use cases are shown: Resolution improvement in one ping measurements, and an alternative way to image Synthetic Aperture Sonar data incoherently.

1 Introduction

Synthetic Aperture Sonar systems (SAS) can be considered a mature field of application in underwater sonar techniques. Data from a number of subsequent pings are combined through signal processing, resulting in increased along-track resolution in SAS-imaging. In practice, the imaging is often performed using the back-projection algorithm. The back projection algorithm is a fast and robust method to solve the inverse problem which gives reliable results. The drawback with back-projection is that it suffers from resolution and ambiguity limitations related to the frequency bandwidth, aperture size and sampling step sizes.

Methods that can extract more information from the available data have been developed in Compressive Sensing (CS). These methods are based on minimizing the l_1-norm of the solution and require that the solution of the inverse problem, in this case the SAS-image, is relatively sparse.

The abstract is organized in the following way: Chapter 2 is a brief introduction to CS and the model used in this work. In chapter 3, two examples showing the utilization of this framework is shown – demonstrating possibilities of these techniques for sonar data.

2 Compressive sensing

Non-stringently expressed, the Nyquist-Shannon sampling theorem states that the sampling rate of a time-continuous signal has to be twice its highest frequency in order to ensure reconstruction. Therefore, it comes as a surprise that, under certain assumptions, it is possible to reconstruct signals when the number of available measurements is smaller than expected based on the Nyquist-Shannon theorem. The underlying assumptions for this is based on that the signal is sparse in a domain, in this case the SAS-image.

A signal is called sparse if most of its components are zero. Another perspective on this is that many signals are compressible, i.e. they can be well approximated by sparse signals. This explains why the family of different compression techniques (such as JPEG, MPEG, or MP3) works so well.

The rise of interest in leveraging CS for signal processing applications has several reasons: a combination of development of theory, faster available algorithms, and faster computers. The field was pioneered by Candés et al in a publication 2004[1]. One early publication reporting an application in magnetic resonance imaging can be found in [2].

2.1 The inverse problem

CS can be applied to several sonar frameworks. In this work, a transmitter sends out a properly designed acoustic signal, the sonar pulse, which is scattered from objects, for example on the sea floor. An array of receivers then measures the acoustic signal resulting from the scattered waves. This can be modelled as an inverse problem:

$$Ax = y, \tag{1}$$

Where the forward operator $A \in \mathbb{C}^{mxN}$, image $x \in \mathbb{C}^{N}$, and measured Sonar signal $y \in \mathbb{C}^{m}$. N and m are the dimensions of the operators. Normally, this problem is underdetermined (*m*<<*N*).

2.1.1 l_1-norm

The problem to find the sparsest solution is formulated to optimize based on the l_0 norm. This is, in general, however an NP-hard problem, therefore the optimization problem is relaxed to l_1-norm:

$$\min \|x\|_1 \ subj.\ to\ \|Ax - y\|_2 \leq \sigma \tag{2}$$

Where the indices 1 and 2 denote the l_1 and l_2-norms, respectively, and σ is an estimate of the noise.

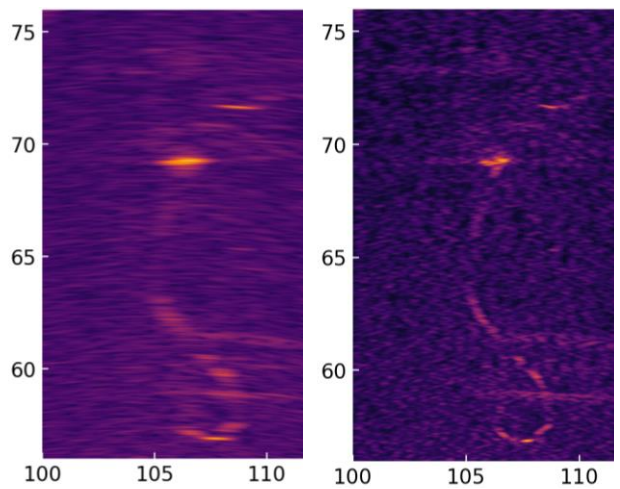

Fig. 1, Enhanced resolution from one ping data. Left image showing delay-and-sum on raw measurement data, right the enhanced image based on CS-results.

In this work, the quadratic constrained l_1-minimization problem has been used, in the SPGL1 implementation (see **Error! Reference source not found.** for more information). For more information regarding this relaxation step, see ref [1].

2.2 Propagators and model

The model is based on isotropic and frequency independent point scatterers.
The back-propagator is the classical delay-and-sum (see for example reference [4]) and the forward propagator is based on the complex wave equation (see for example reference [5]).

3 Examples

The measurements are collected using the experimental platform Sapphires in the freshwater Lake Vättern. Sapphires has a side-looking sonar array giving a SAS resolution of <4x4 cm, using conventional back-projection. Results using the method described above are presented, both from one ping data and from a combination of several pings. All measurements are covering the same object, a rope loop, to simplify comparisons. The images show $\|x\|_2^{1/3}$, to decrease the dynamics in the results.

3.1. High resolution from one ping measurements

Enabling a higher resolution from one ping measurements than given by the beam pattern for a linear array of a certain length is (still) a highly interesting field. The resolution is significantly enhanced by using the model described above together with the l_1-minimization. This is visualized in Fig. 1, where the image to the left shows the image using the delay-and-sum formulation on the measurement data. The image to the right shows the CS result. The CS results have been achieved using the results from the minimization together with the forward- and backwards-propagator for a synthetically extended array (but keeping the inter-distance between the receivers). Worth mentioning is that the sparsity in this case is around 10%.

3.2 Multiple pings

This example is based on data from three different pings, with no overlapping elements, to demonstrate the possibilities of constructing SAS-images. No autofocus techniques were used in this example, i.e. no correcting phase factor for incorrect sound velocity or other sources of errors. Therefore, the images from the different pings are incoherently added. It is clearly seen, see Fig. 2 (left without CS, right with CS), that the resolution is enhanced.

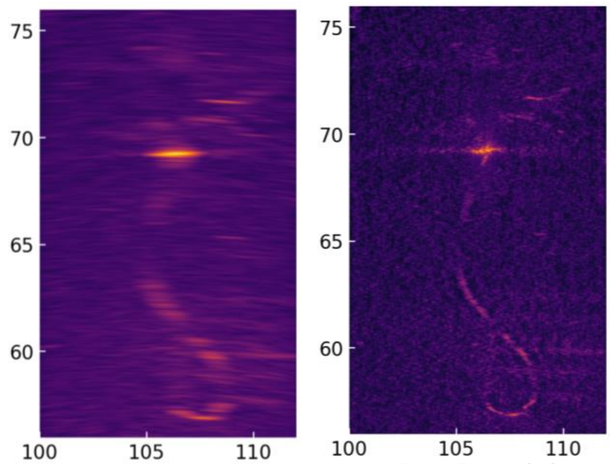

Fig. 2, Comparison between incoherently added data.from three non-overlapping pings without (left) and with CS(right)

References

[1] E.J. Candés, J. Romberg, T. Tau, IEEE Trans. Inform. Theory, 52 489-509 (2004)
[2] M. Lustig, D. Donoho, J. M. Pauly, Magn. Reson. Med., 58 1182-95 (2007)
[3] E. van den Berg, M. P. Friedlander, SIAM J. on Scientific Coputing, 31(2): 890-912 (2008)
[4] R. E. Hansen, *Introduction to Synthetic Aperture Sonar*, *Sonar System*, Prof. Nikolai Kolev (ed.) (2011), ISBN: 978-953-307-345-3
[5] R. J. Urick, *principles of underwater sound*, 3rd edition (1983), ISBN: 9780932146625

Author/Speaker Biographies

Dr. **Andreas Gällström** is a part of the sonar and sensors department and a specialist at Saab Dynamics in Linköping, Sweden. He also holds a position as visiting researcher at Lund University. His research interests are in sonar and underwater acoustics. He has published more than 30 journal and conference papers. He received the M.Sc. and Ph.D. degrees in physics from Linköping University in 2005 and 2015, respectively.
Ms **Louise Rixon Fuchs** is a systems engineer at the sonar and sensor department at Saab Dynamics, Linköping. Since 2017 she is an industrial PhD student at the Royal Institute of Technology in Stockholm at the division of Robotics Perception and Learning. Her research interests are in sonar perception, signal processing and machine learning. She received the M.Sc. degree in Electrical Engineering from the Royal Institute of Technology in 2019.
Dr. **Christer Larsson** is the manager for radar sensors and a senior specialist at Saab Dynamics in Linköping, Sweden. He also holds the position as adjunct professor at Lund University. His research interests are in RCS, radar imaging and in the electromagnetic properties of materials. He has published more than 50 journal and conference papers. He received the B.Sc. and Ph.D. degrees in physics from Uppsala University in 1981 and the Royal Technical Institute in Stockholm in 1990, respectively.