SAAB

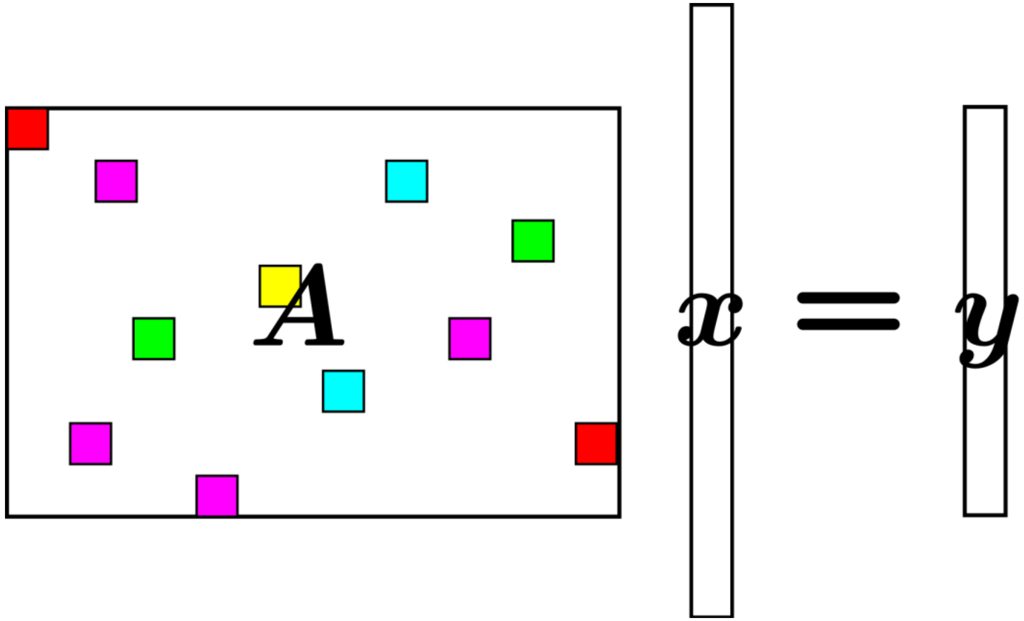
Enhancing Sonar resolution through smart signal processing

UDT 2019 Sthlm

A. Gällström. L. Fuchs, C. Larsson

Outline

- Compressive Sensing
  - The Inverse Problem
  - l_1-norm
  - Propagators
  - Model
- Examples
  - High Resolution from 1 ping measurement
  - Scatterer point representation
  - Multiple pings
- Summary



H. Nyquist (1889-1976) and C. Shannon(1916-2001)

Nyquist-Shannon Sampling Theorem

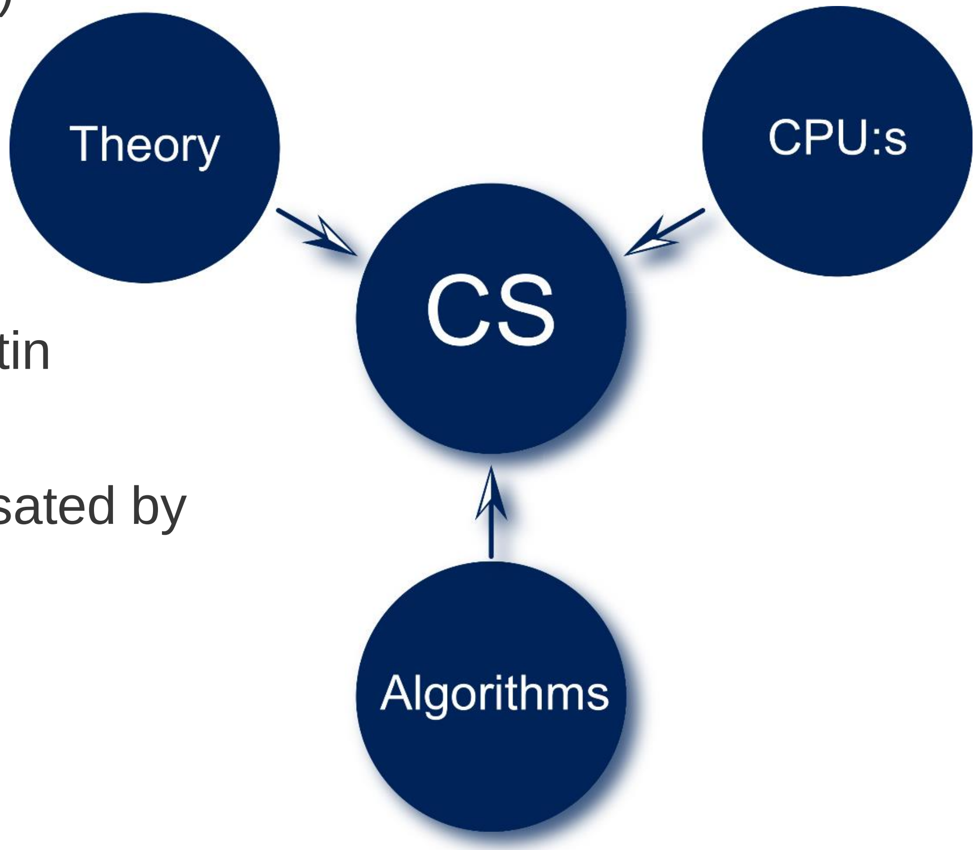
”If a function contains no frequencies higher than B Hertz, it is completely determined by giving its ordinates to a series of points spaced 1/(2B) seconds apart.” (Wikipedia)

Data compression

- **Example:** JPEG compression from 487 to 71 kB (16%)
- Typical compression rate with a factor of 10
- To much data is collected
- **Idea:** Reduce data collection and compensate with signal processing

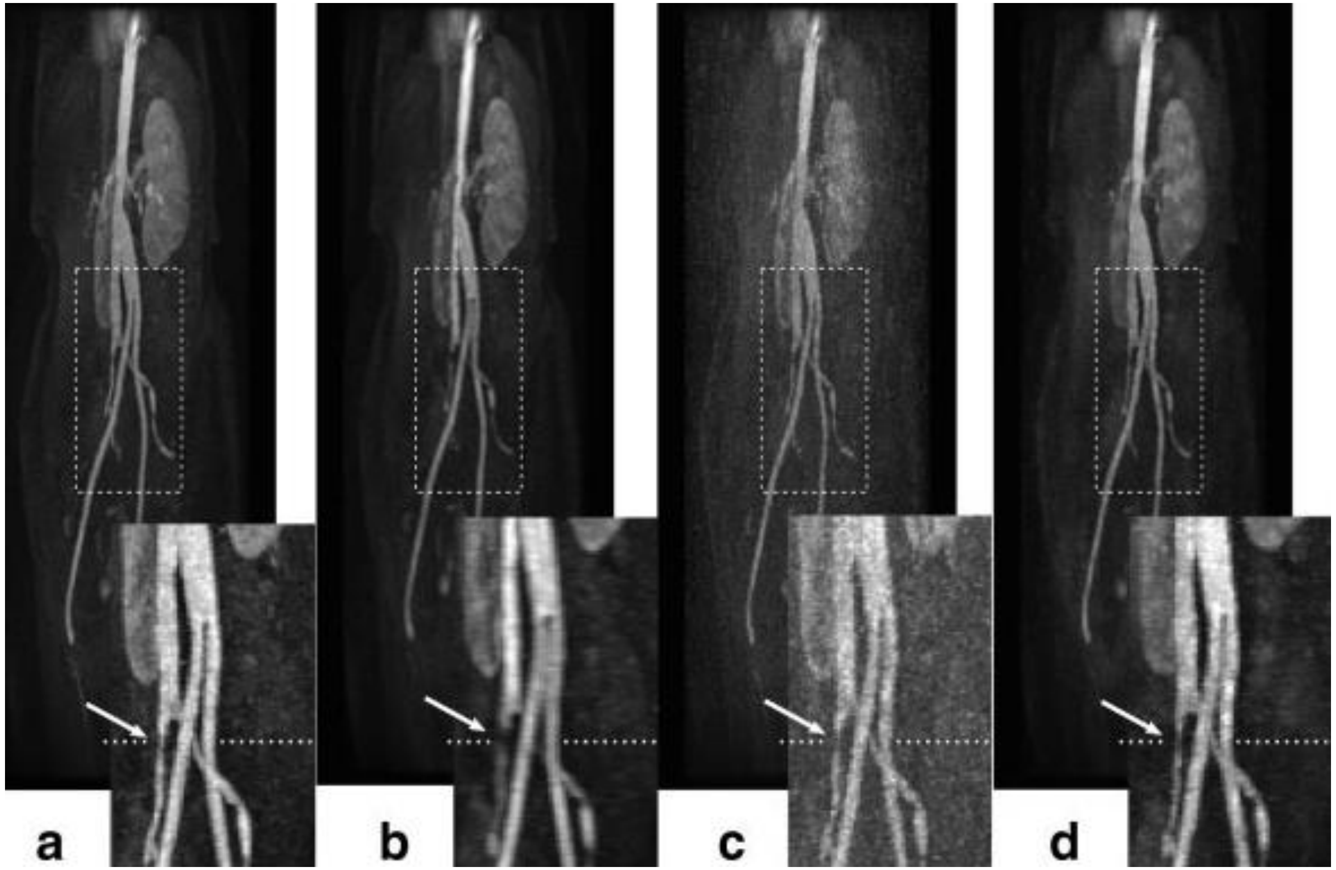
Compressive Sensing

- Developments of theory for Compressive Sensing (CS)
- Faster algorithms
- Faster computers (flops/cpu)
- Enabling practical use of Compressive Sensing (CS)
- Pioneered by: Emmanuel Candés, David Donoho, Justin Romberg and Terence Tao (2004)
- CS means that less data is collected which is compensated by using postprocessing



Compressive sensing – early application MRI

- Magnetic resonance imaging (MRI)
- Picture a shows an MRI-image using complete data set and conventional data processing
- Picture d shows an image using 20% of data set (from a) and CS



M. Lustig, D. Donoho, and J. M. Pauly. "Sparse MRI: The application of compressed sensing for rapid MR imaging." Magnetic resonance in medicine 58, no. 6 (2007): 1182-1195.

Inverse problems

- Linear set of equations:

$$Ax = y \begin{cases} x \in \mathbb{C}^N \\ A \in \mathbb{C}^{m \times N} \\ y \in \mathbb{C}^m \end{cases}$$

- y is an observation/measurement, and we are trying to find x (parameter)
- Normally this set of equations are undetermined (m<<N) =>infinitely many solutions (provided that there exists at least one)
- Sonar: The reflected signal is used to determine position, speed, target class…, i.e. parameters.

Inverse problems

- Underdetermined linear set of equations:

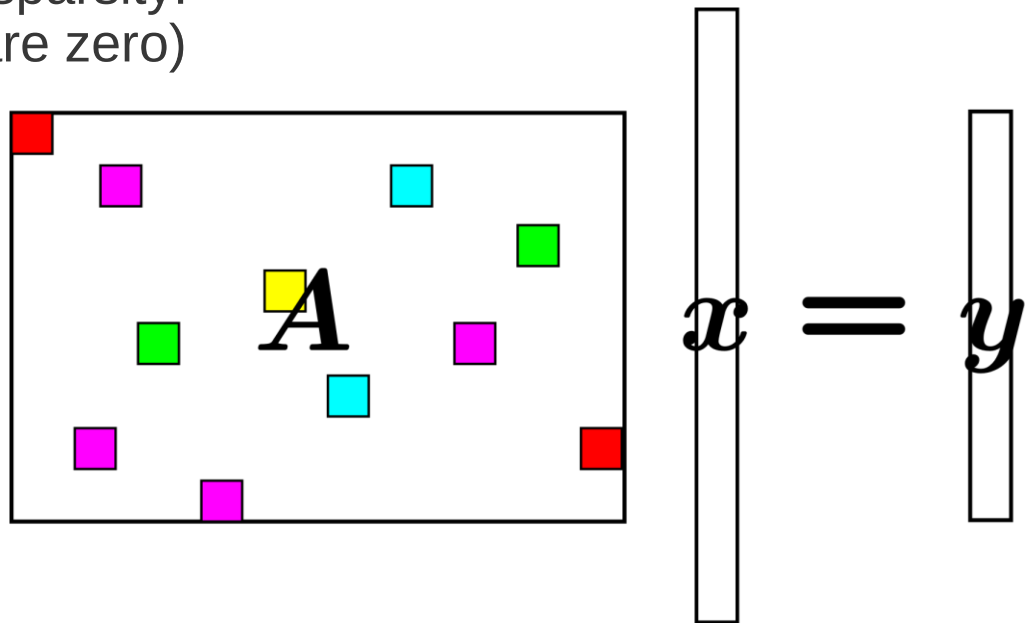
$$Ax = y$$

- Possible to reconstruct signals under assumption of sparsity!
(A vector/matrix is sparse if most of its components are zero)
- Efficient algorithms exists, in this work:
Quadratically constrained l1-minimization problem:

$$\min\|x\|_1 \ subj.to \ \|Ax - y\|_2 \leq \sigma$$

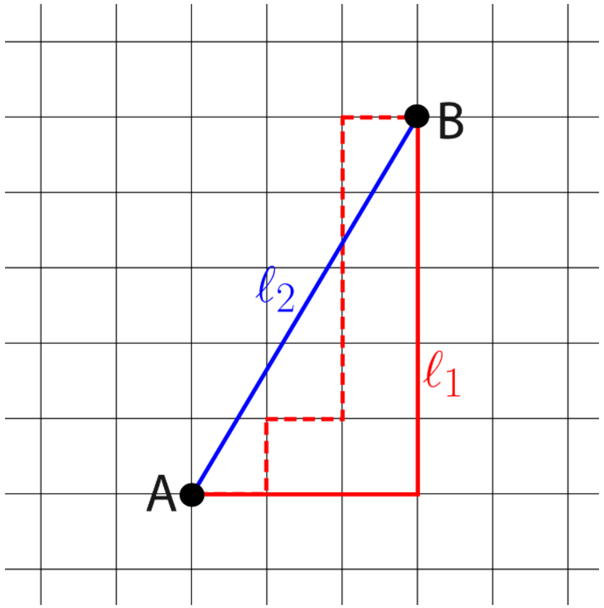
σ related to SNR

(other variants exist: LASSO, Dantzig selector, …)



l_0-, l_1- and l_2-norms

- Norm: total size or length
- l_2: "straight-line" – Euclidian distance $\|x\|_2 = \sqrt{\sum_i x_i^2}$
- l_0 : Sparsity – $\|x\||_1 = \#(i|x_i \neq 0)$ (total number of non-zero elements in a vector. Useful for finding the sparsest solution. However: minimization is regarded as NP-hard.
- l_1: $\|x\|_1 = \sum_i |x_i|$
- $l_1$relaxed l_0:used in Compressive sensing. Not as smooth as l_2, but this problem is better and more unique than the l_2-optimization.
  The optimization road is convex optimization.

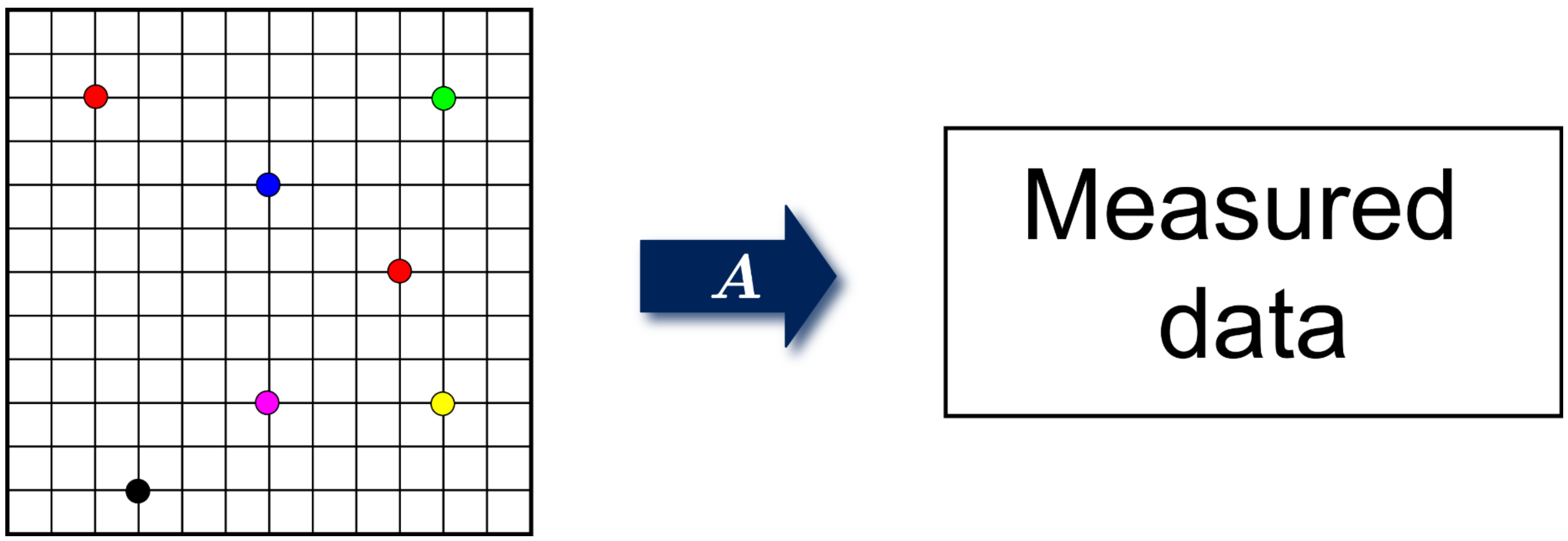


Compressive sensing

- Sonar
  - Point scatter model
  - Back-propagator
  - Forward-propagator

Model (Point scatterer)

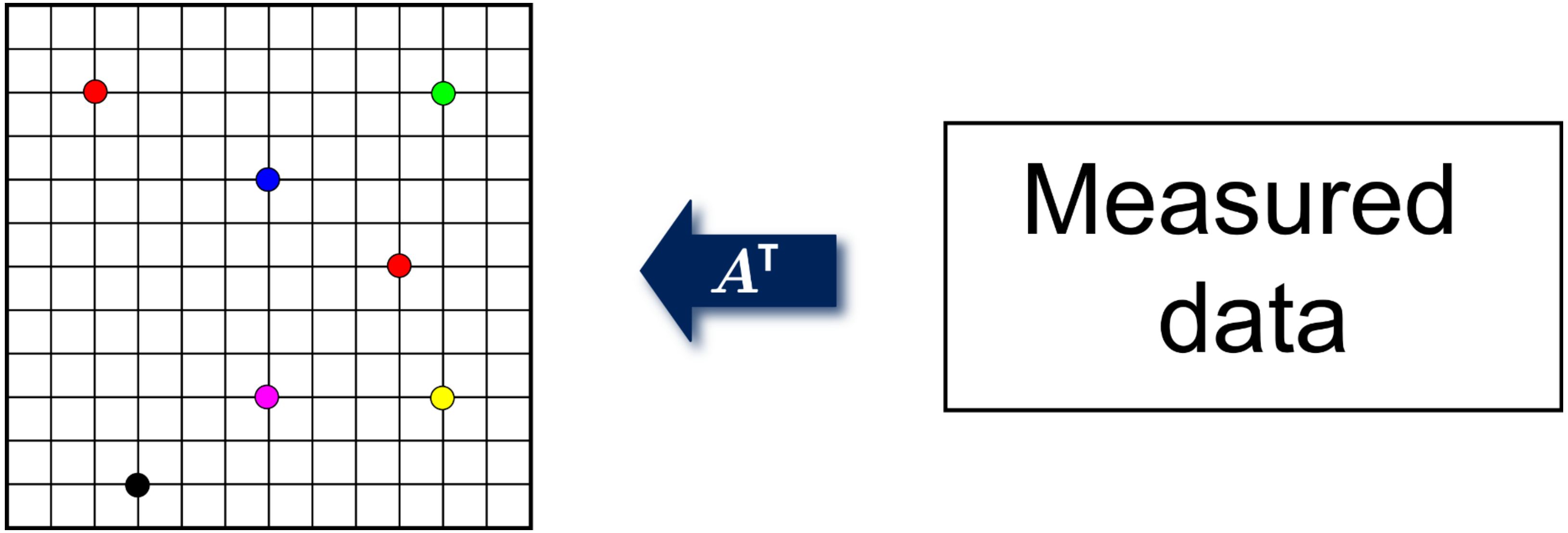
- Isotropic, frequency independent point scatterer as model.
- $Ax = y$
- A: signal generator (from point scatterer to element signals)



Back-propagator

- Classical Delay-And-Sum:

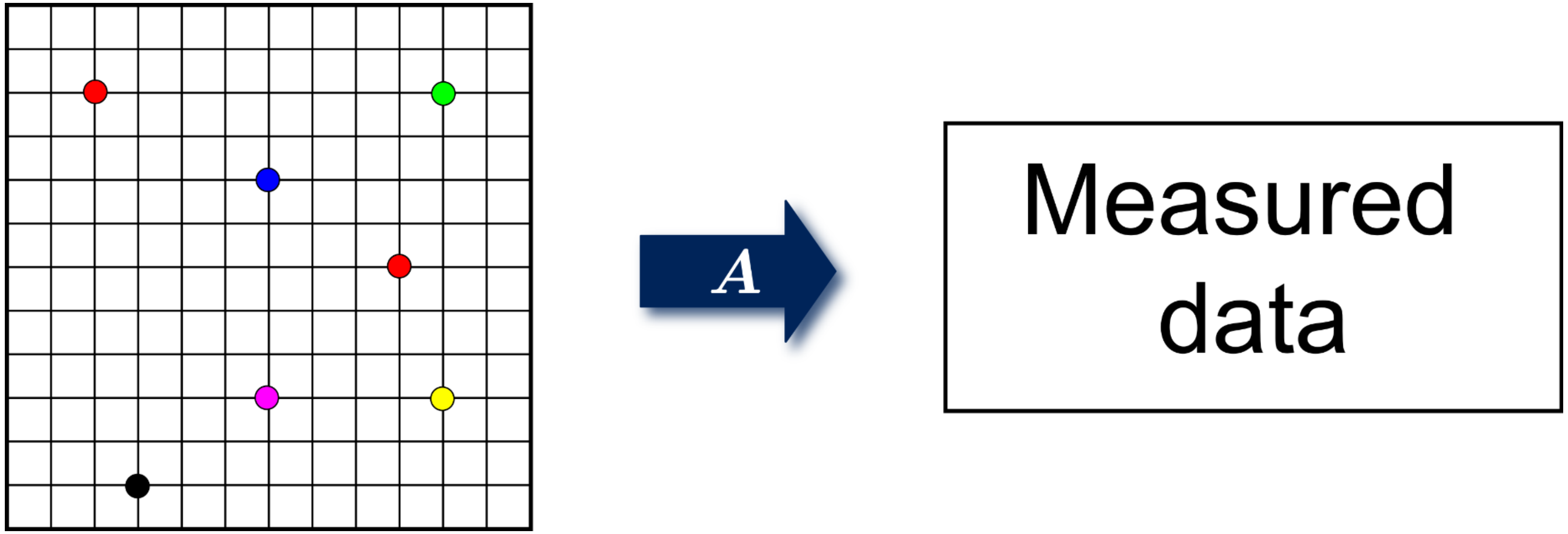
$$\hat{\gamma}(r) = \frac{1}{N}\sum_{n=1}^{N} A_n s_n\big(t_n(r)\big)$$



Forward-propagator

Signal observed at time t and position r emitted from a point scatterer at r':

$$s(t,r) = \frac{A\left(t - \frac{|r - r'|}{c}\right)}{|r - r'|^2} e^{i\omega t\left(t - \frac{|r - r'|}{c}\right)}$$

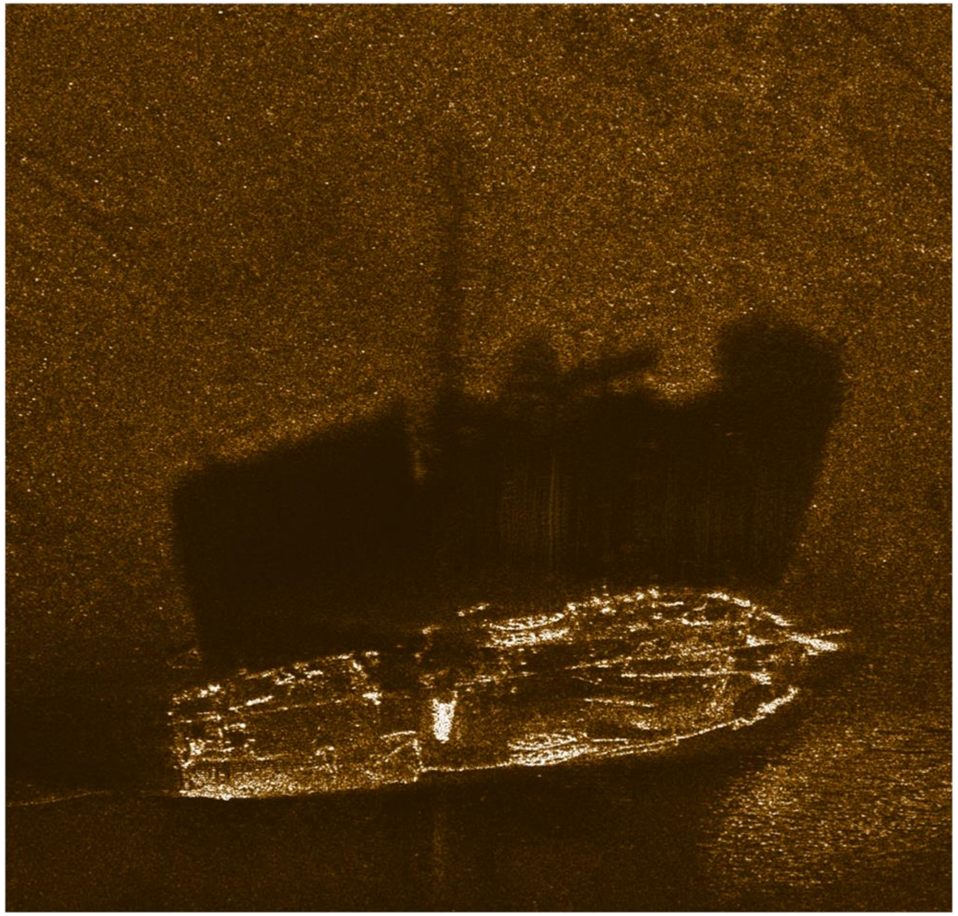


Outline

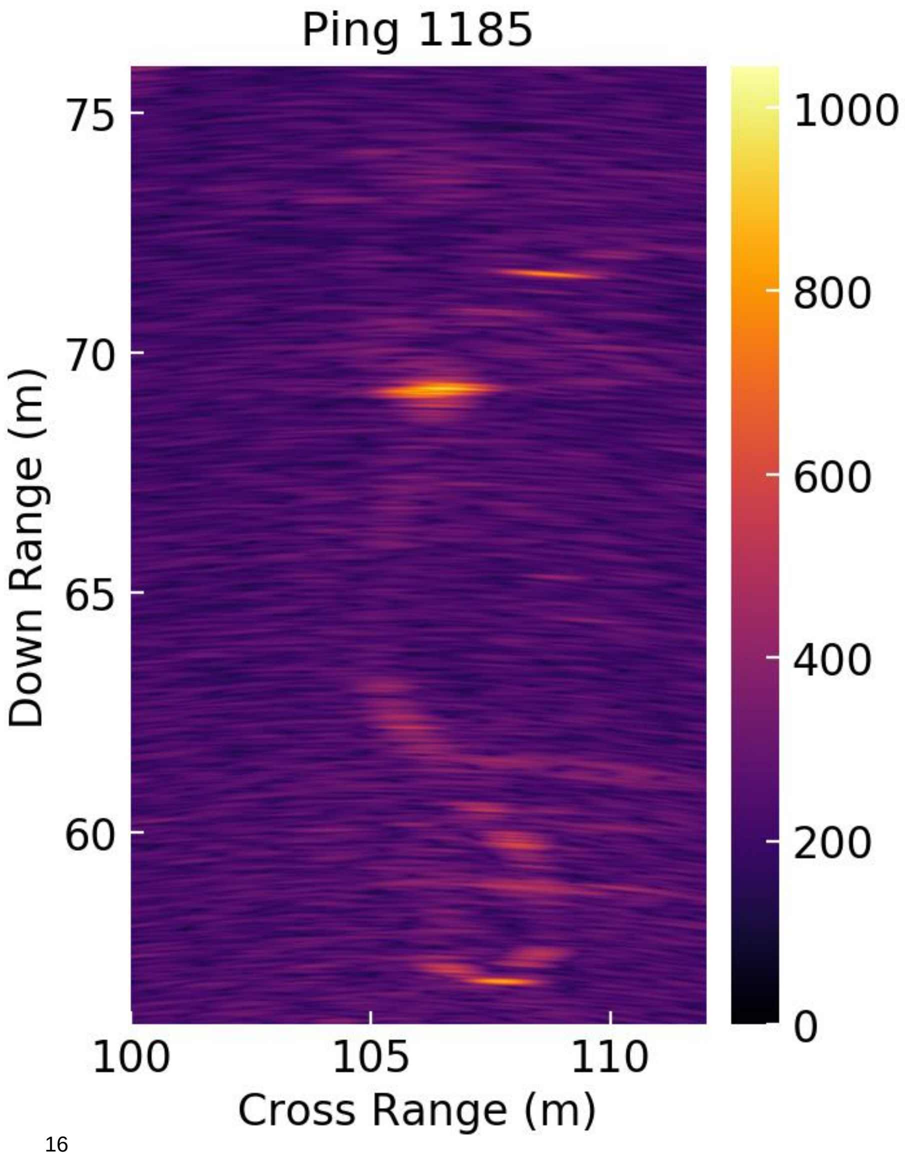
- Synthetic Aperture Sonar
- Compressive Sensing
- Examples
  - High Resolution from 1 ping measurments
  - Robustness
  - Modell from different pings
  - Autofocus – position based
  - Autofocus – phase based
- Summary

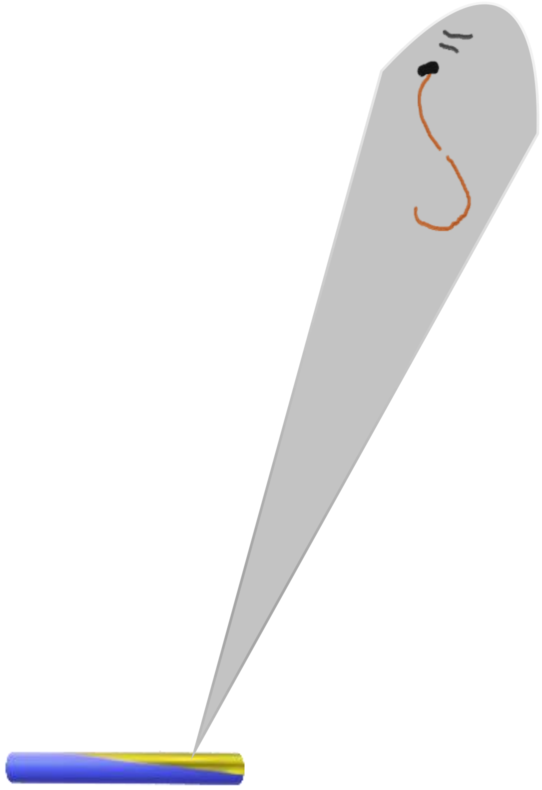
Measurement setup

- Sapphires
- SAS resolution <4x4 cm
- Fresh water lake: Vättern

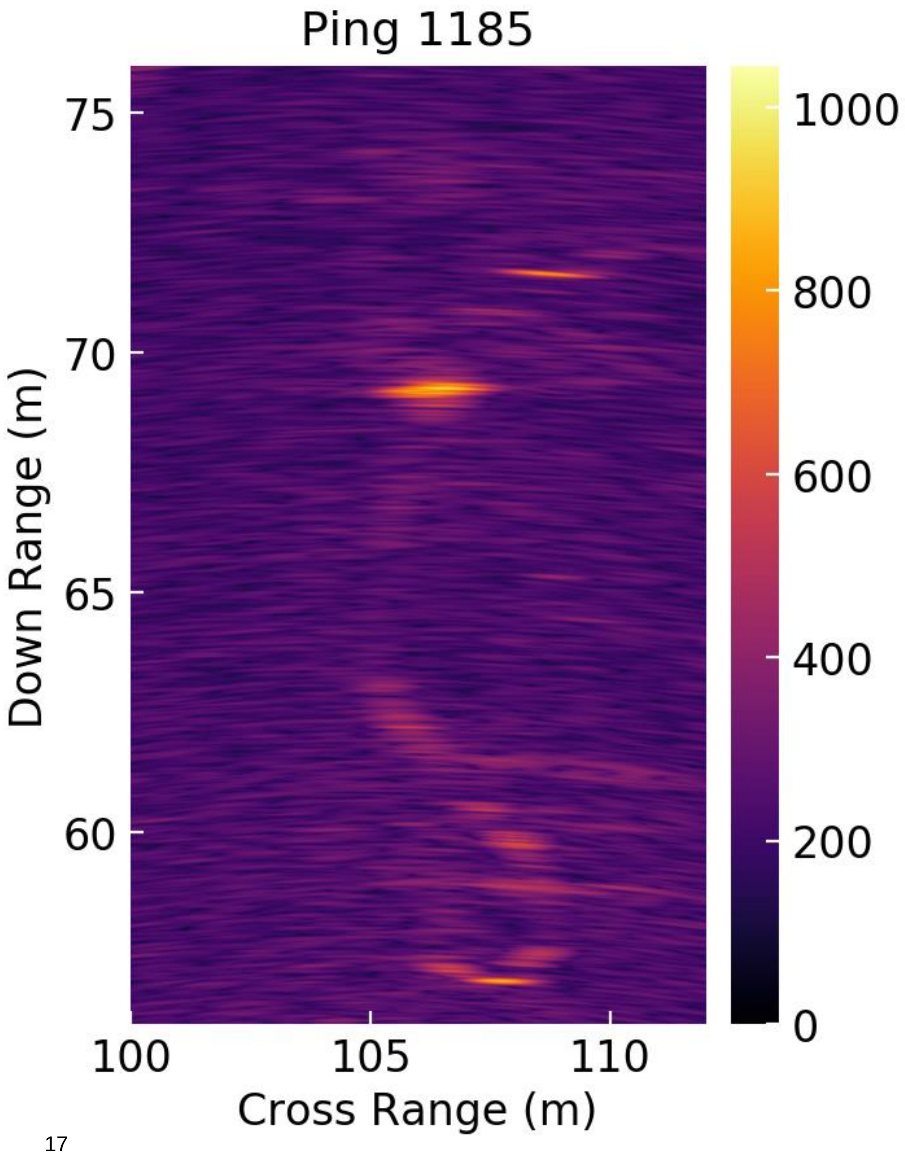


Normal resolution from 1 ping measurement



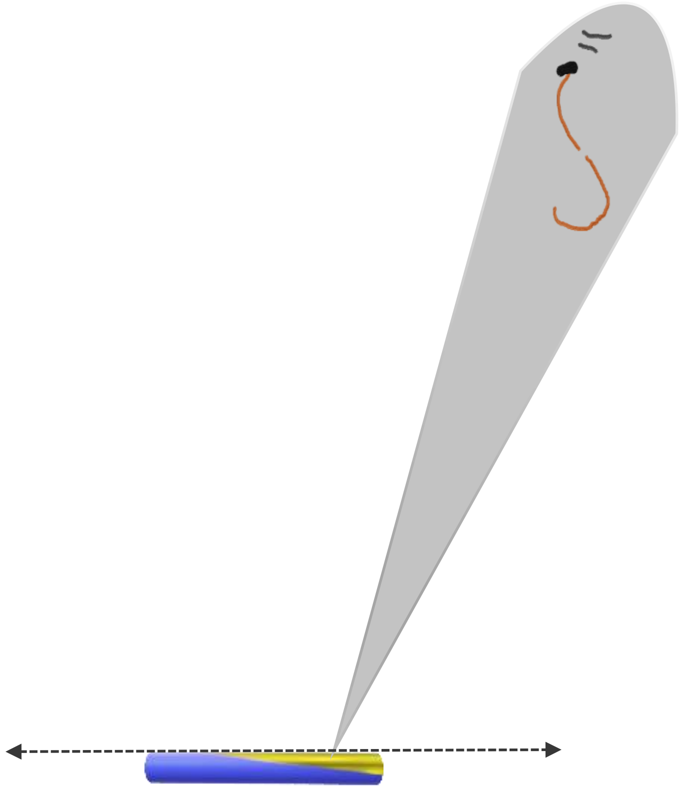


Normal Resolution from 1 ping measurment



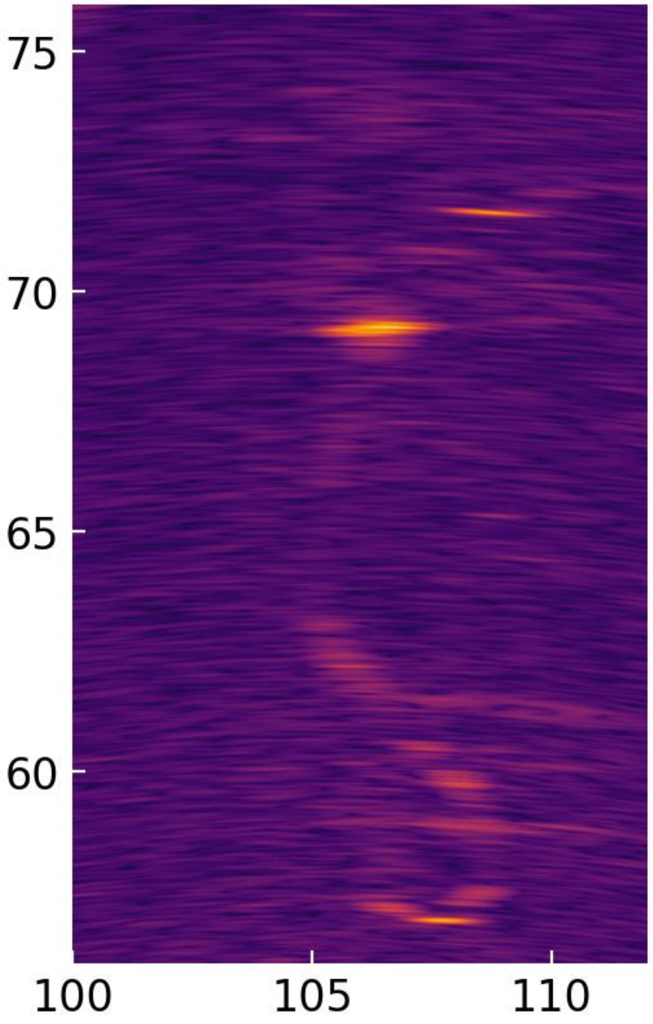
$\min \|x\|_1 \; subj.to \; \|Ax - y\|_2 \leq \sigma$

Visualize with longer array

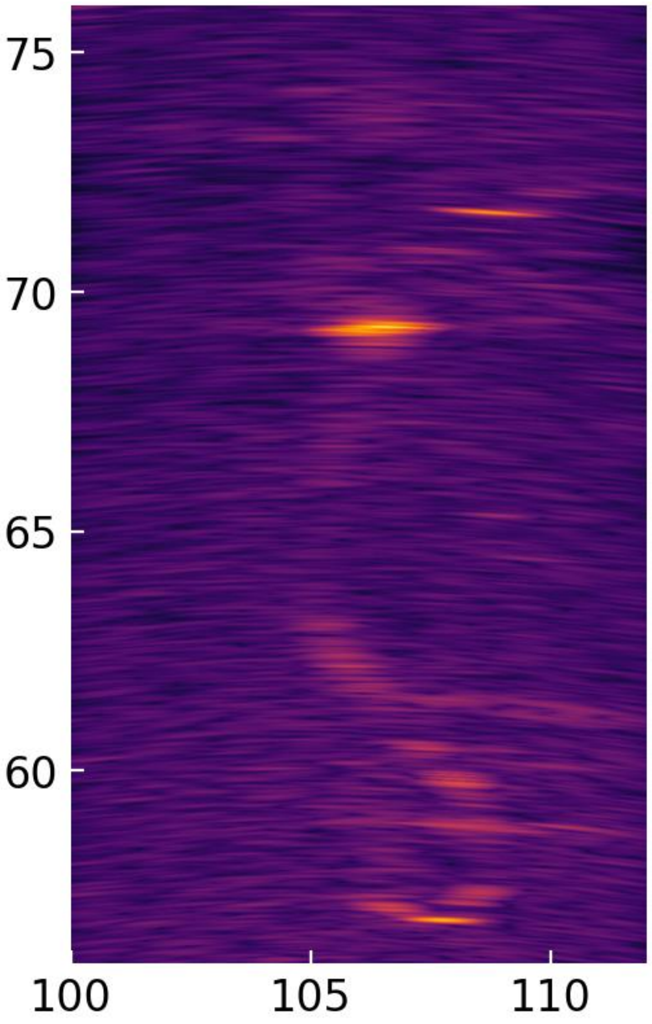


Enhanced resolution

$\min\|x\|_1 \; subj.to \; \|Ax - y\|_2 \leq \sigma \;\; same\ resolution$

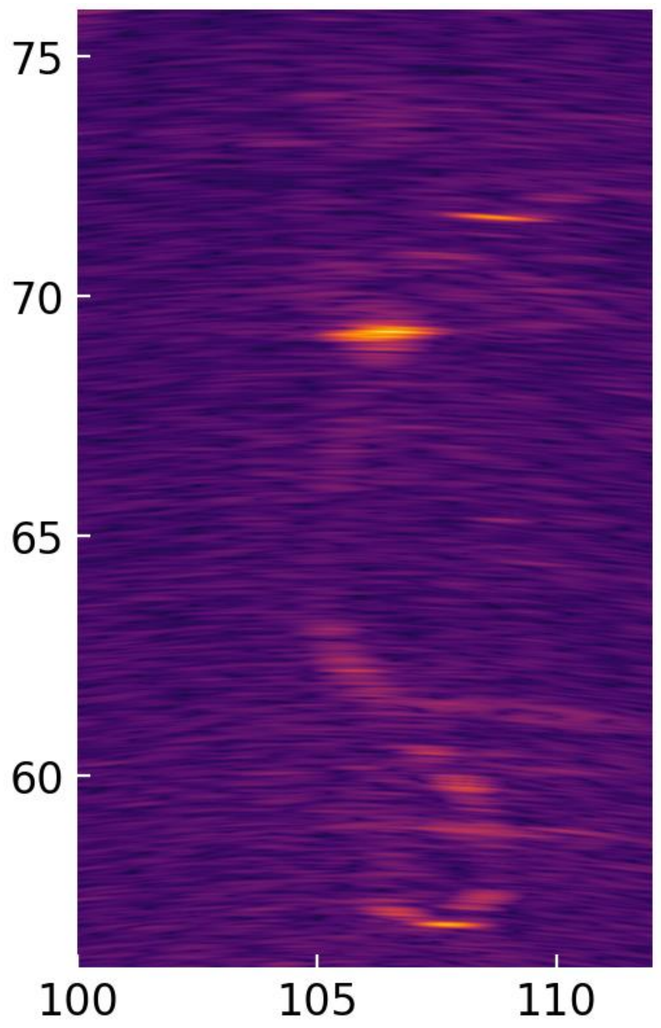


Same data used for both this images

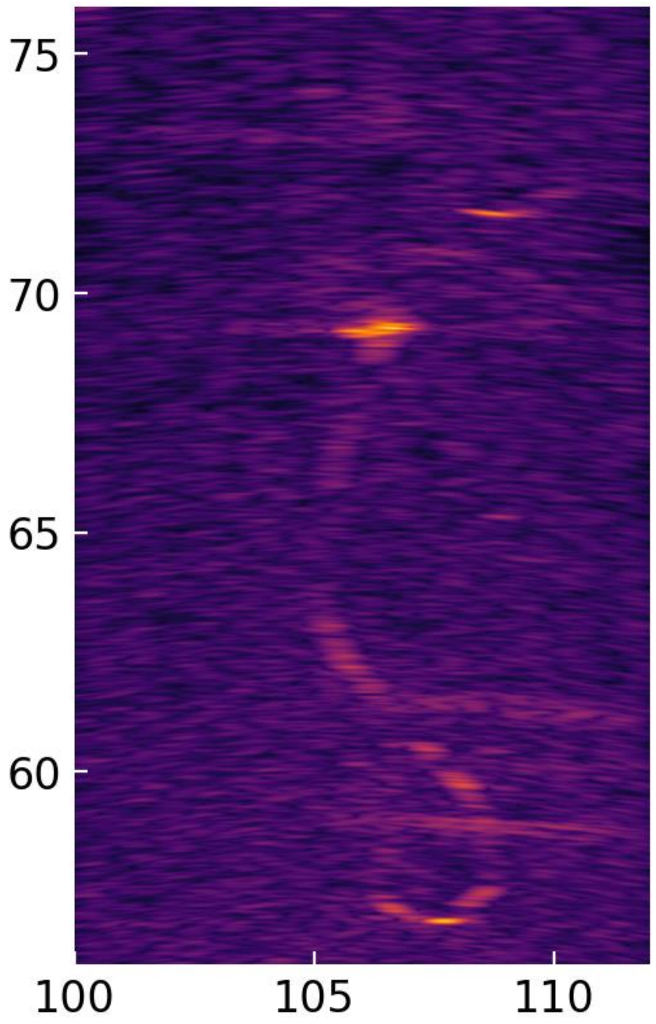


Enhanced resolution

$$\min\|x\|_1 \; subj.to \; \|Ax-y\|_2 \leq \sigma \;\; res: x2$$

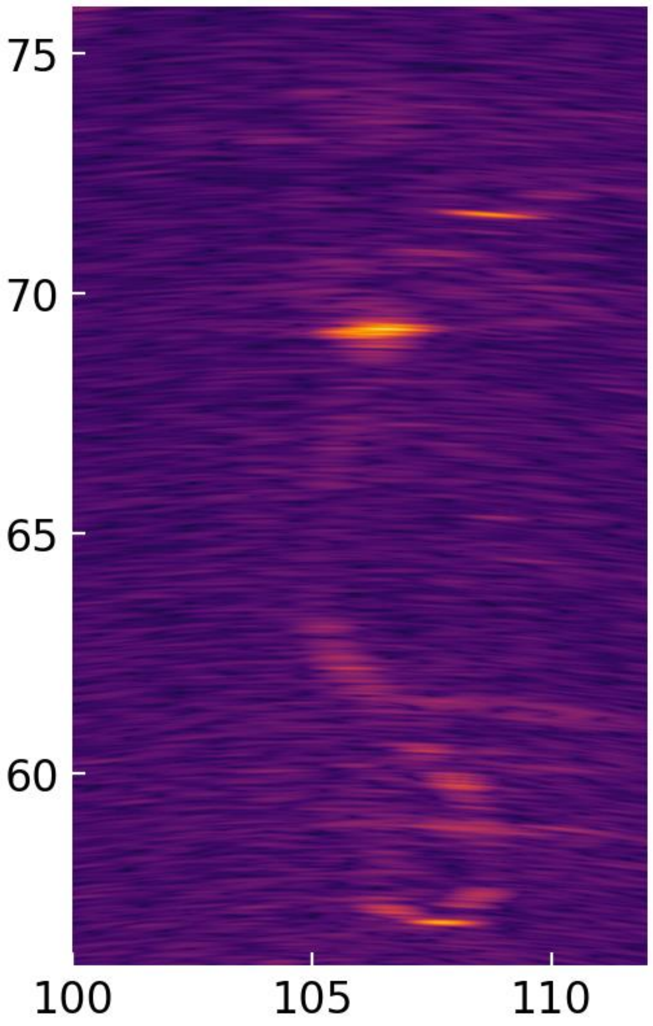


Same data used for both this images

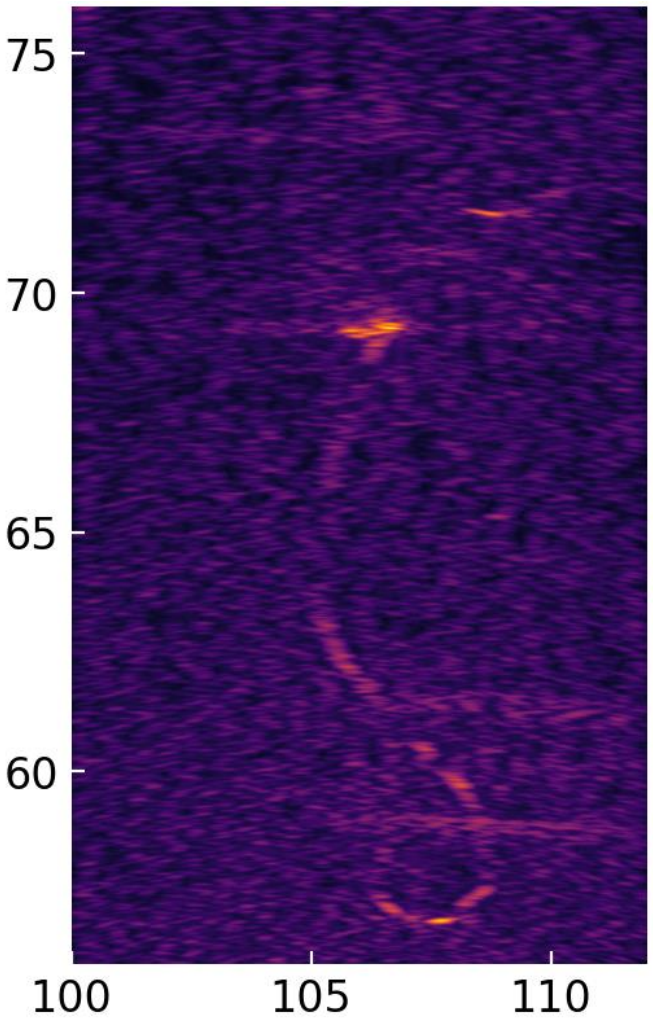


Enhanced resolution

$\min\|x\|_1 \; subj.to \; \|Ax - y\|_2 \leq \sigma \;\; res: \; x4$

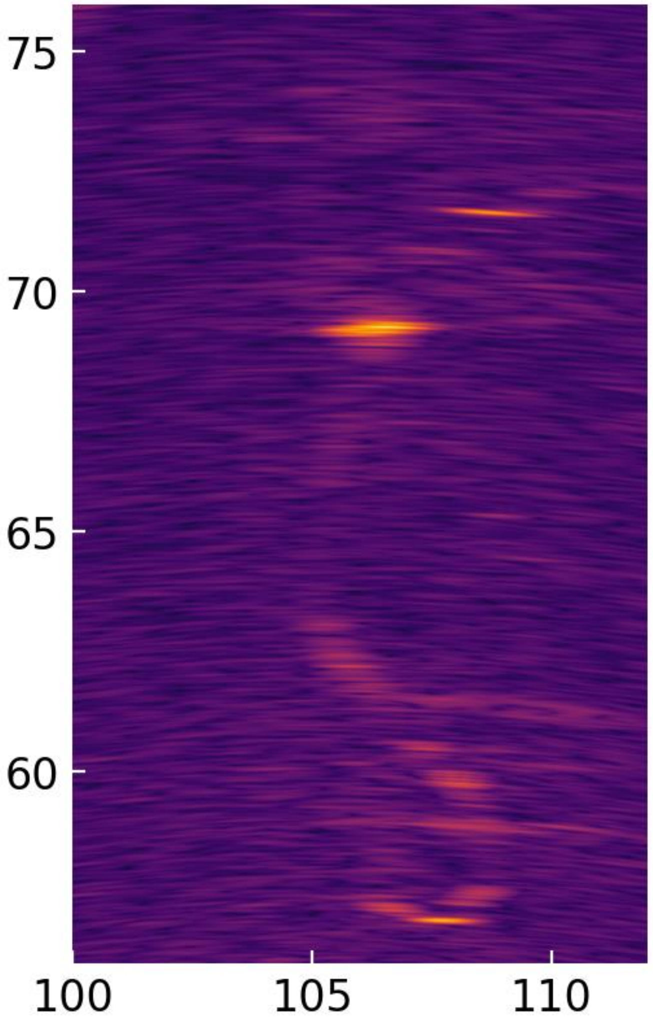


Same data used for both this images

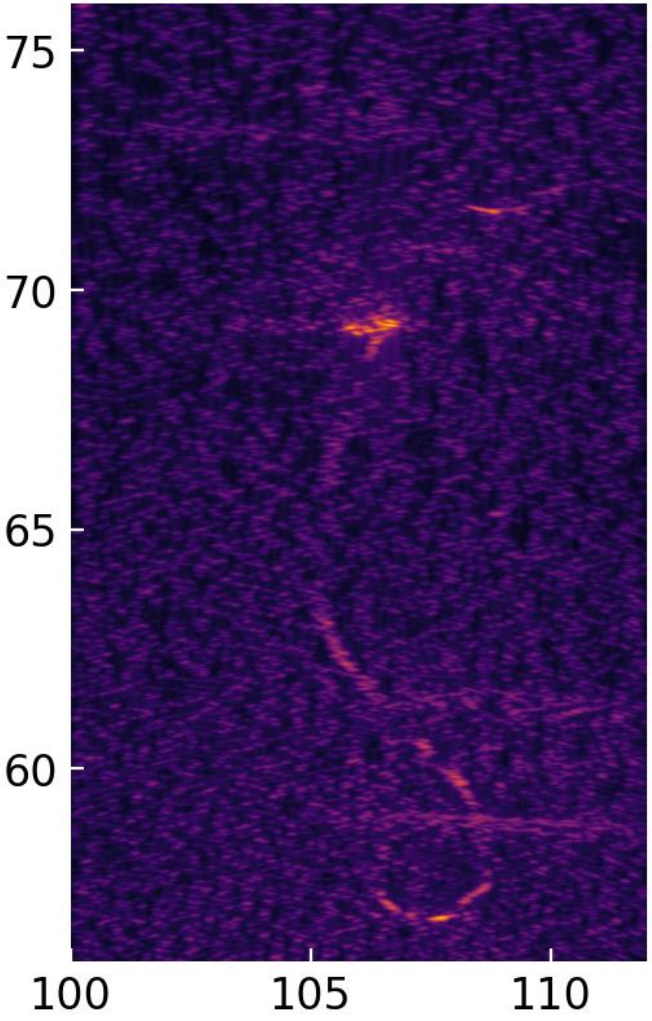


Enhanced resolution

$\min \|x\|_1 \; subj.to \; \|Ax - y\|_2 \leq \sigma \;\; res: x8$

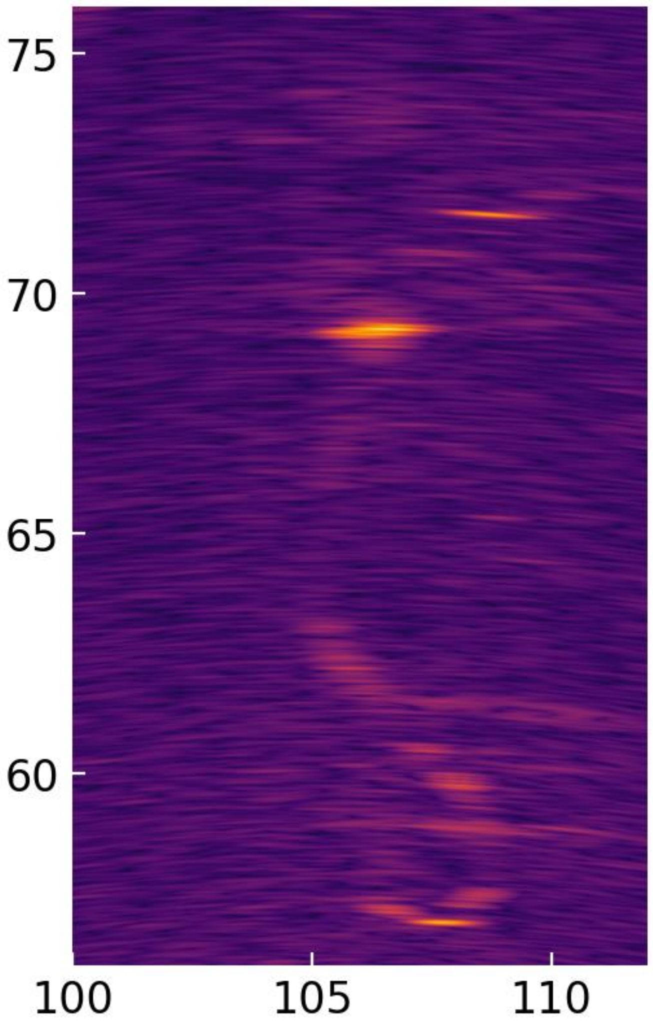


Same data used for both this images

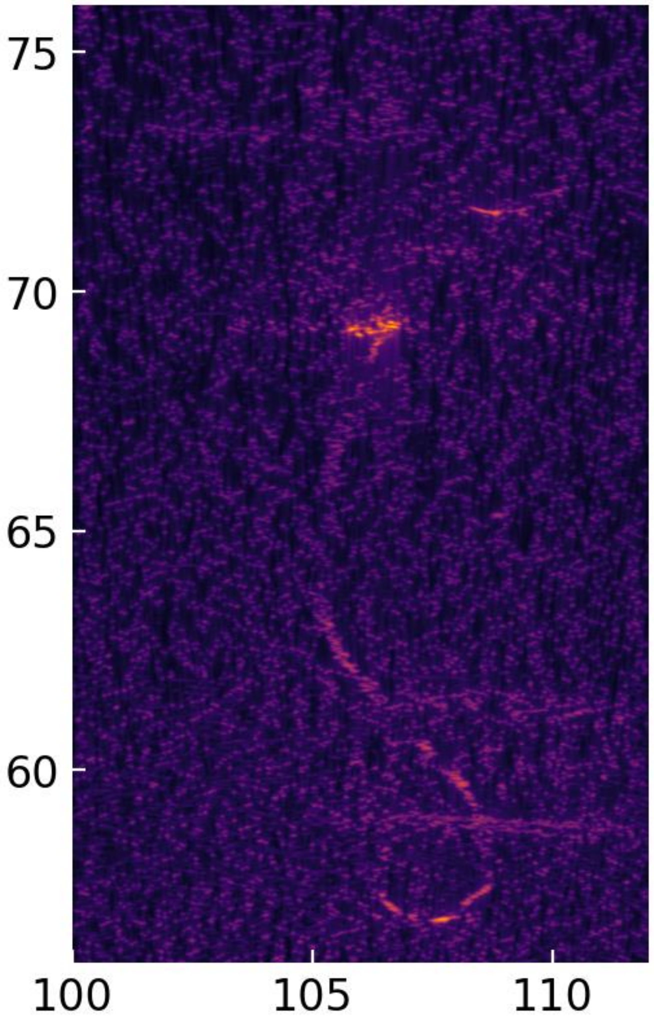


Enhanced resolution

$$\min\|x\|_1 \; subj.to \; \|Ax - y\|_2 \leq \sigma \quad res\!: x16$$

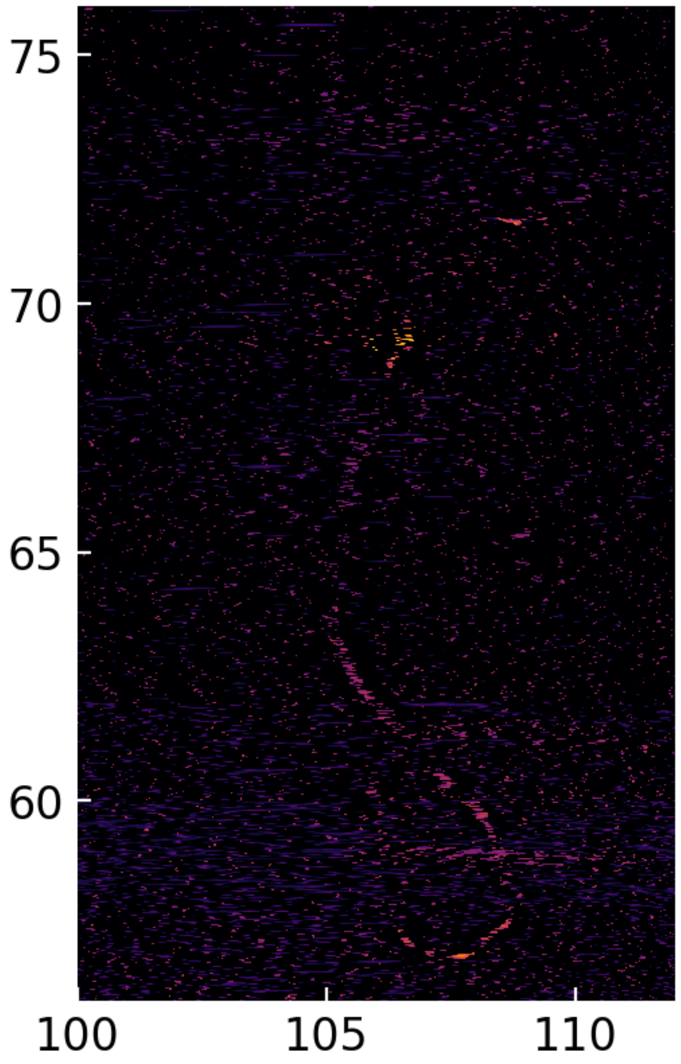


Same data used for both this images



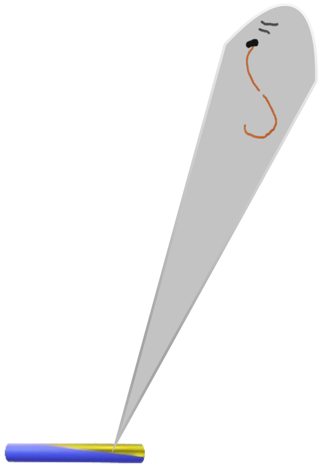
Modell

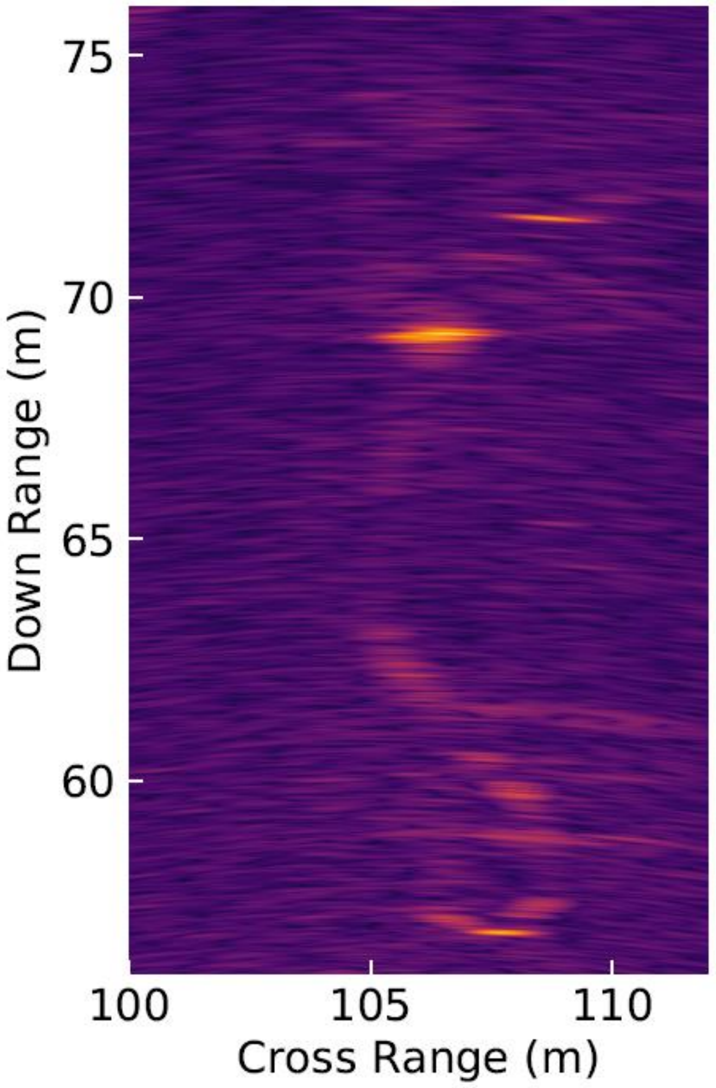
- Visualization of point scatterers based on one ping
- Sparsivity: ~10%



SAS

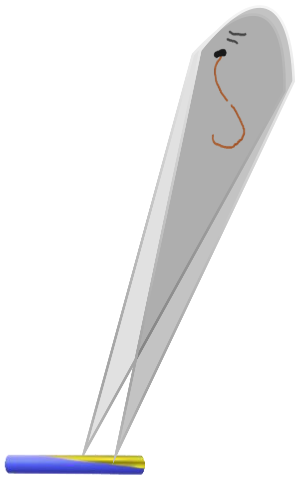
1 ping

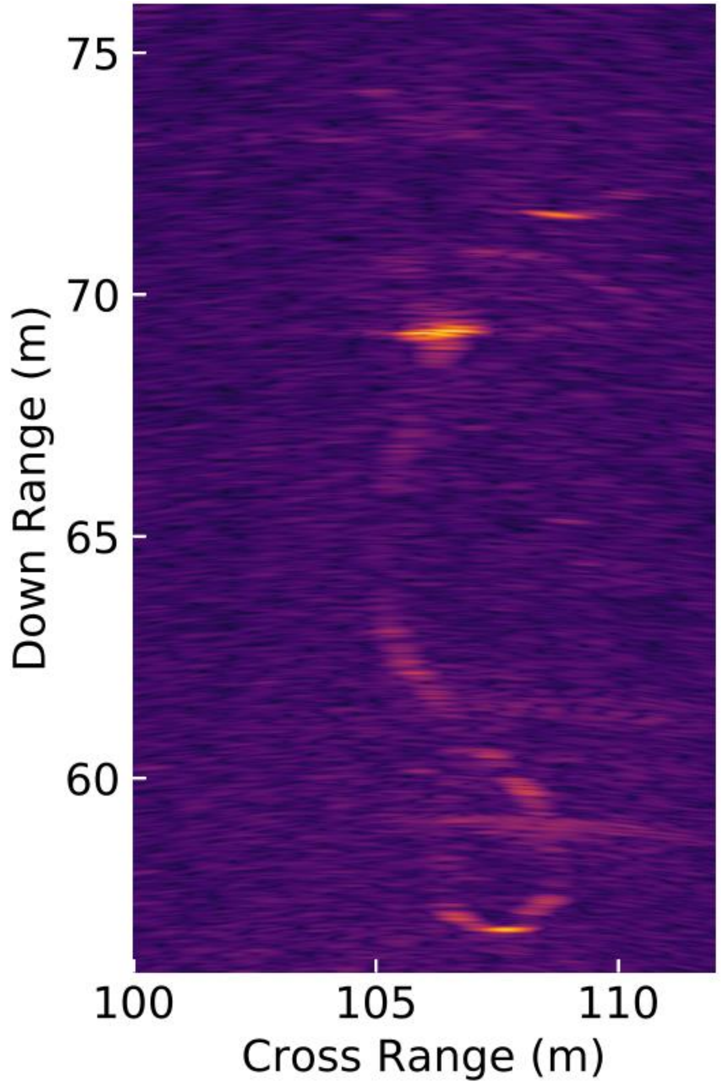




SAS

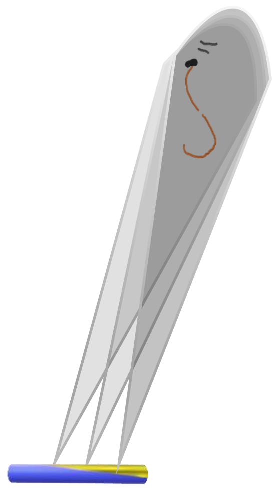
2 pings

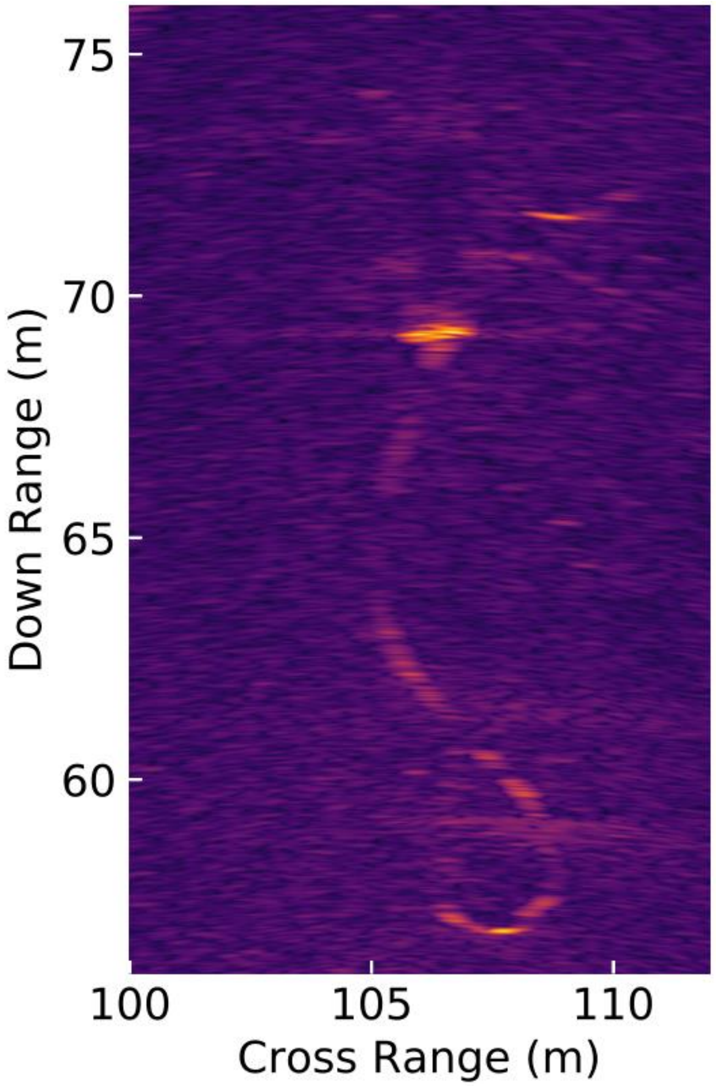




SAS

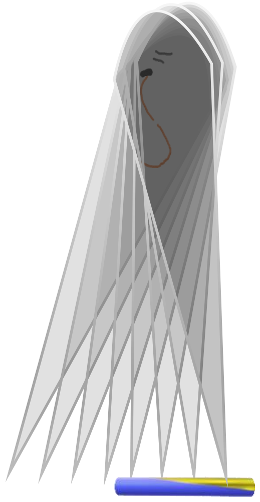
3 pings

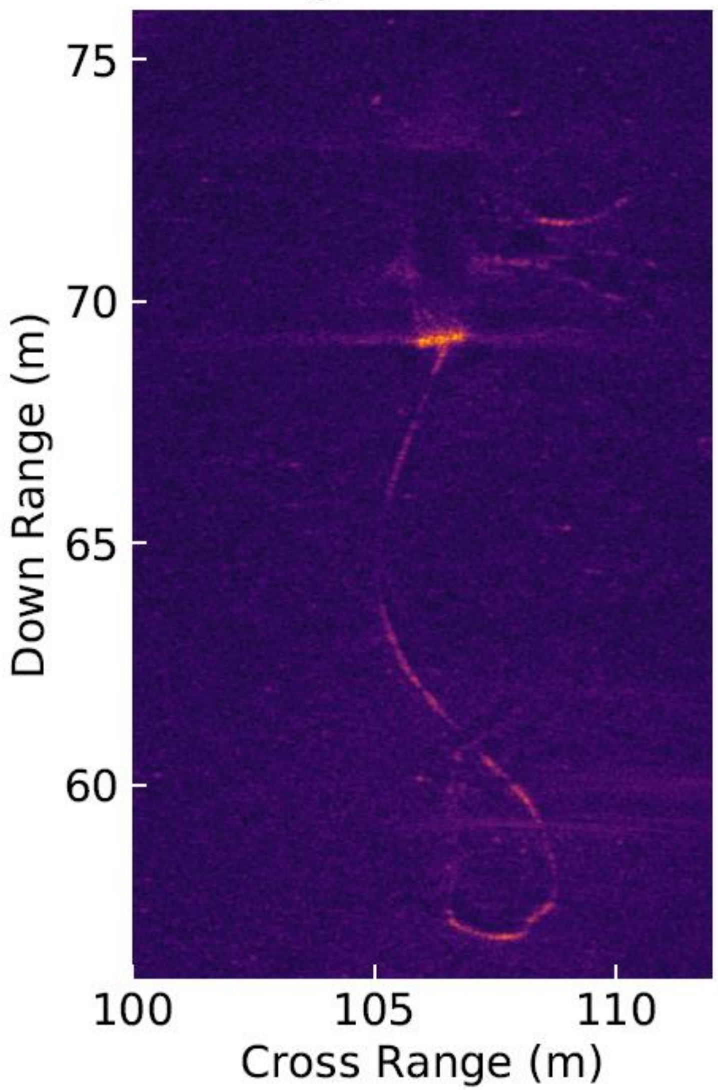




SAS

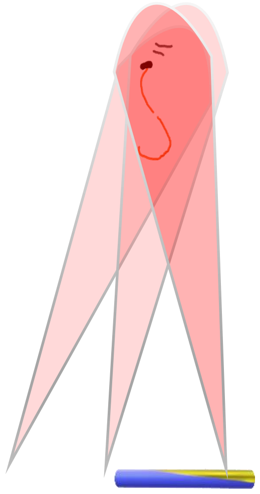
~30 pings

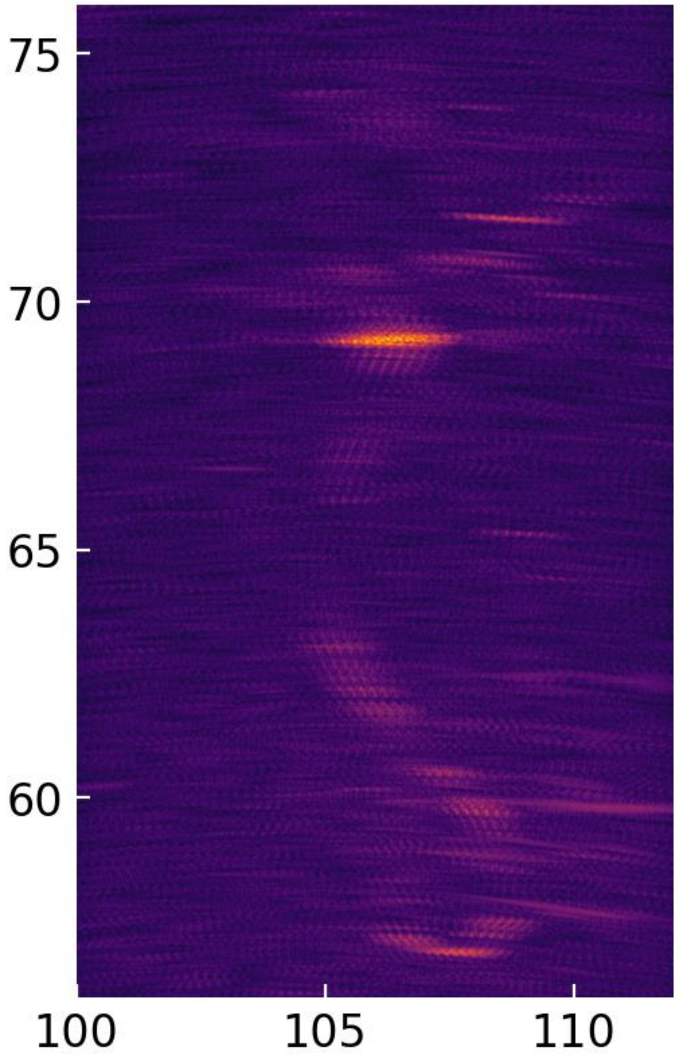




Several pings

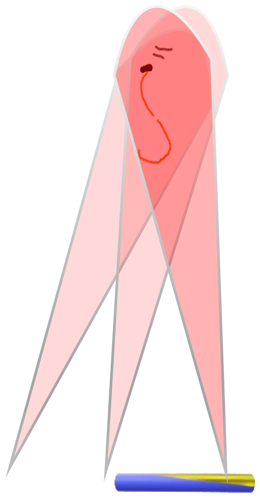
Three pings used, with no overlap (and no autofocus), *coherently* added

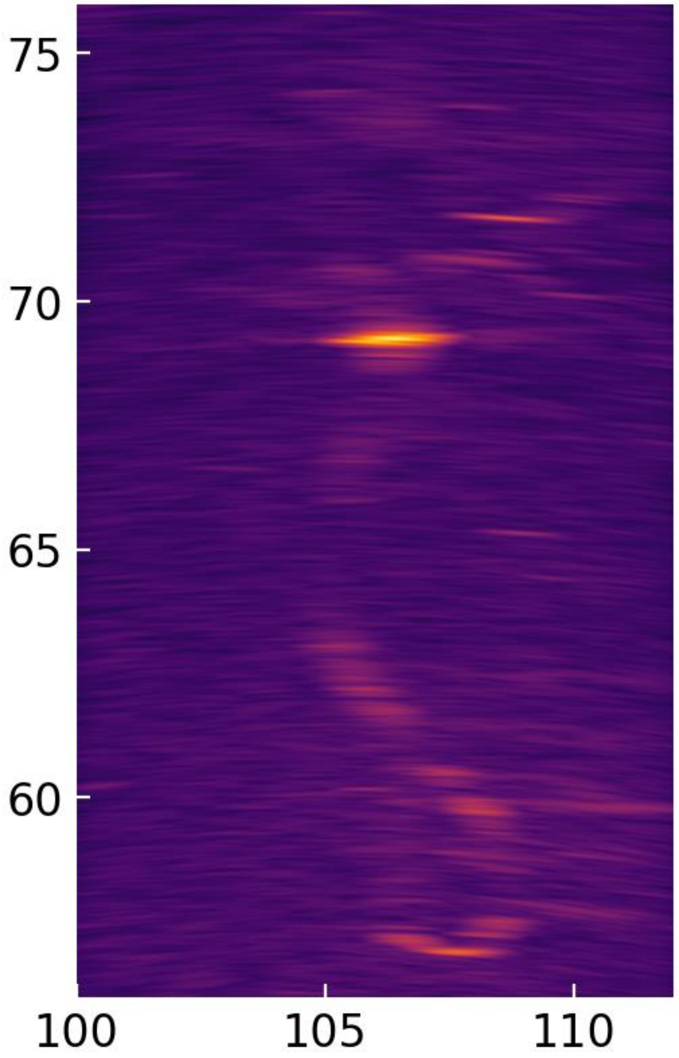




Several pings

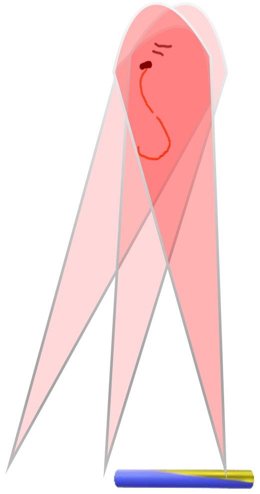
Three pings used, with no overlap (and no autofocus), *incoherently* added

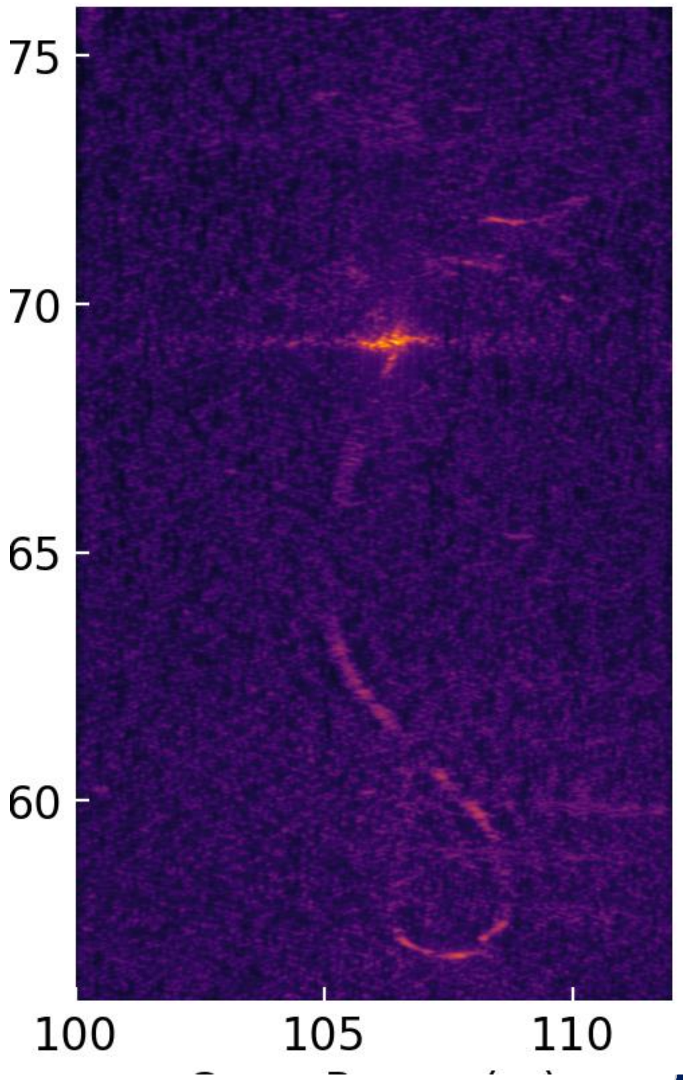




Several pings

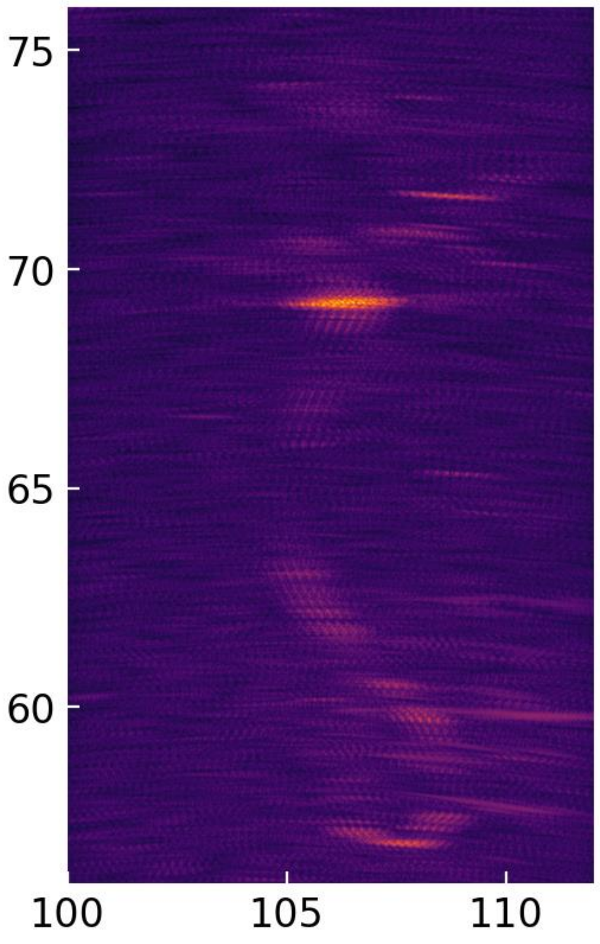
Three pings used, with no overlap (and no autofocus), *processed* using CS and incoherently added



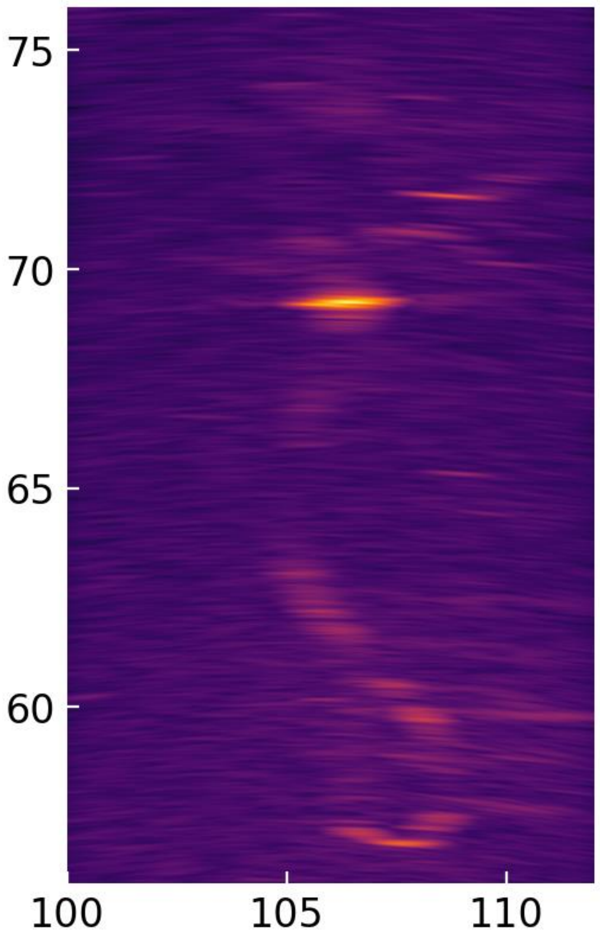


Same data from 3 non-overlapping pings without autofocus

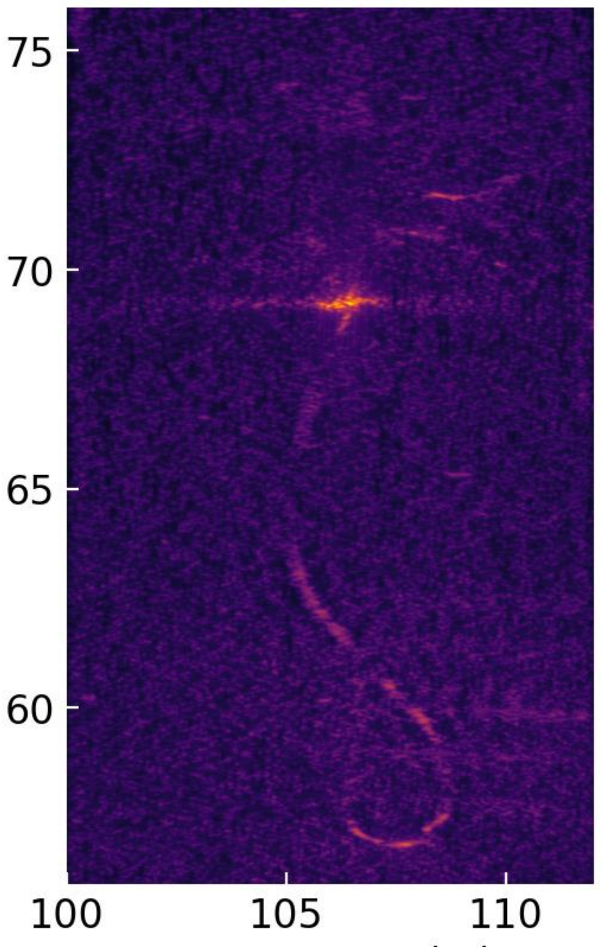
Coherently added



Incoherently added



CS and *incoherently* added



Summary

- Compressive sensing utilizing the sparsity in sonar data is an interesting and promising tool

- Examples:
  - Enhancing resolution in one-ping images
  - Combining multiple pings

Thank you