Analysis of the vibro-acoustic behavior of a submarine hull on a wide frequency range using experimental and numerical approaches

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Abstract — Acoustic stealth is one of the main operational capabilities for a submarine vehicle. To meet always stricter requirements in terms of radiated noise, it is important for the shipbuilder to be able to predict the noise at the early design stages. One of the key elements is the transfer function between a mechanical excitation on the pressure hull of the submarine vehicle and the far-field pressure radiated into water. This transfer function is commonly called p/F. A dedicated numerical method, called the Condensed Transfer Function (CTF) method, has been developed to calculate the vibro-acoustic behavior of a submarine hull. The method is based on a substructuring approach, where a model of an infinite fluid-loaded cylindrical shell is coupled to models of internal structures such as stiffeners and bulkheads. The fluid-loaded cylindrical shell is described analytically, while the internal structures are modeled by the finite element method (FEM). For each uncoupled subsystem, mechanical admittances are calculated at the junctions between the subsystems, and approximated with a set of functions called condensation functions. The interactions between the subsystems are estimated using the superposition principle, the force equilibrium and the displacements continuity. Compared to fully analytical models, the CTF method has more flexibility on the geometry of the internal frames. Compared to models completely described by the FEM, the present approach has lower calculation cost. Consequently, transfer functions can be calculated on a wide frequency range (from some Hz to several kHz) with limited calculation costs. Besides, an experimental campaign has been conducted to measure the transfer function p/F of a scaled model of the pressure hull. Based on laser vibrometer measurements on a model in air, the radial acceleration are measured at the surface of the shell. The far-field radiated pressure is calculated through the stationary phase theorem to deduce the transfer function p/F. This approach has two main advantages. First, no microphones are needed to estimate the radiated noise from the shell and second, the directivity of the radiated pressure can be studied. In this paper, the principle of both the numerical and the experimental approaches are described. A test case is presented and the results using both approaches are compared. The directivity pattern and some physical features of the vibro-acoustic behavior of the hull are discussed.

1 Introduction

Predicting the vibro-acoustic behaviour of a submerged vessel is important for the naval industry in order to meet strict requirements in terms of radiated noise. More particularly, one is interested in the transfer functions between the pressure radiated in the far field and a point force exerted on the hull, commonly called p/F. These transfer functions depend on the frequency but also on the locations of the point force and of the observation point.

In the early stages of the design of a submarine, two main approaches are possible to estimate the radiation from a shell: numerical models and experimental studies on scale models. The two approaches can be used jointly for a better understanding of the physical phenomena. Nowadays, numerical approaches are more widely used than experimental approaches because of reduced costs. Moreover, numerical tools allow to easily conducting parametric studies. Many numerical methods are available to solve vibro-acoustic problems. In the low frequency range, the preferred approaches are discretization methods such as the FEM (Finite Elements Method) and the BEM (Boundary Elements Method) [1]. These methods consist in solving the partial differential equations on discrete meshes. The geometrical complexity of a system can be precisely modelled but the methods induce high computational costs when the frequency increases because of the need of refined meshes. In the high frequency range, statistical methods such as the SEA (Statistical Energy Analysis) are preferred [2]. In this approach, a system is divided into several subsystems between which power can be exchanged. The SEA method is only valid at high frequencies, where the systems are modally dense. It is computationally efficient but only give global (space and frequency averaged) results.

In the naval industry, there is a need to fill the gap between the low frequency and the high frequency range. This gap, for a typical submarine application is for frequencies between several hundreds of Hertz and several kilohertz. A hybrid method, called the CTF (Condensed Transfer Function) method has been developed to tackle this particular issue. The CTF method is a sub-structuring method, where the cylindrical shell is described by an analytical approach, and the internal structures such as stiffeners are modelled by FEM. It gives the advantage to be computationally efficient while allowing describing accurately the internal structures.

Experimentally, the transfer function p/F can be measured by exciting a scale model with a shaker, and measuring the radiated pressure at a location in the surrounding fluid with a hydrophone. However this approach is limited because only one transfer function can be measured at the time. Using a scanning laser vibrometer and the stationary phase theorem can overcome this limitation by estimating the radiated pressure at all the locations around the cylindrical shell.

In this paper, the CTF method and the experimental procedure are described for the case of a cylindrical shell. The methods are applied on a scale model and results are compared. Physical phenomena are discussed.

2 The Condensed Transfer Functions method

In the CTF method, a set of orthogonal functions called condensation functions are used to approximate mechanical admittances at line junctions between coupled subsystems [4]. In this study, the CTF method is applied to a stiffened cylindrical shell, as shown in Fig. **1**. The junctions correspond to circumferential rings where the cylindrical shell is coupled to stiffeners. The displacements of the shell are calculated using analytical equations of motion coupled with the Helmholtz equation in the fluid. The displacements and acoustic pressures are converted from the spatial domain to the wave-number domain by Fourier transform. The wavenumber domain is discretized and the equations of motion are solved for couples of axial and circumferential wavenumbers. The admittances of the stiffeners are calculated using FEM, without taking fluid loading into account. The cylindrical shell is clamped at both ends. The coupling forces exerted by the stiffeners on the cylindrical shell at are calculated using the admittances of the uncoupled subsystems [5]. The equations of motion of the shell under the coupling forces are also calculated in the wavenumber domain. The total response of the cylindrical shell due to the point force and the coupling forces yields the radial acceleration of the infinite cylindrical shell coupled to the stiffeners. The far-field radiated pressure is obtained using the stationary phase theorem [6].

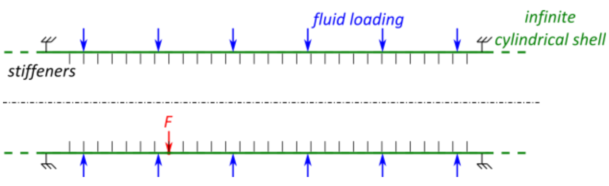

Fig. 1. Schematic view of the CTF model

3 The experimental procedure

The experimental procedure takes place in a semi-anechoic room (80 Hz cut-off frequency). A stiffened cylindrical shell is hanged on a rotating platform, as shown in Figure 2. A shaker excites the shell in the radial direction. A scanning laser vibrometer is placed at a fixed position and measures the radial acceleration of the surface of the cylindrical shell [3]. An array of 6 microphones is placed at 1 m far from the cylindrical shell to measure the radiated pressure. The procedure consists in measuring simultaneously the vibrations on a vertical line and the radiated pressure for a position of the cylinder, before rotating the cylindrical shell and the shaker by 9°. The system being symmetric, it is sufficient to measure between 0 and 180°. A 2D Fourier transform along the spatial coordinates is done to express the radial accelerations in the wavenumber space. The radiated pressure all around the cylindrical shell is deduced with the stationary phase theorem [6].

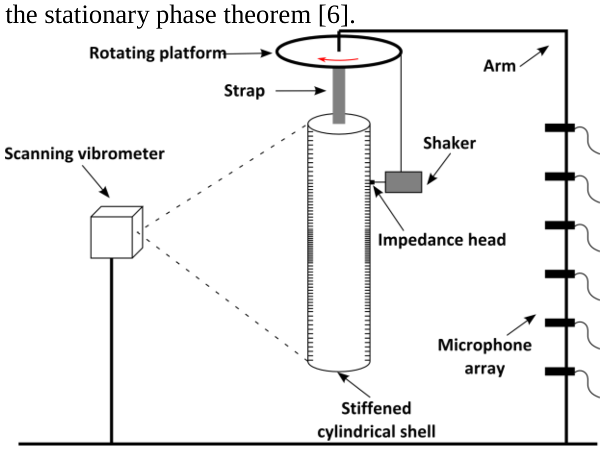

Fig. 2. Sketch of the experimental procedure (from [3]).

4 Test case description

A steel cylindrical shell is considered. It is 1500 mm long, 1.5 mm thick and with a radius of 100 mm. It has 72 stiffeners of rectangular cross-section 5x1 mm², divided in five sections with three different stiffeners spacing. The shell is closed by two 15 mm thick end caps. The shaker excites the shell at a distance of 305 mm from the top of the cylindrical shell.

Fig. 3. Pictures of the cylindrical shell hanging on the arm (left) and seen from the inside (right).

The quantities of interest are the radial vibrations of the shell and the radiated power. These quantities are measured according to the experimental procedure described in paragraph 3, and predicted using the dedicated CTF method. The measurements are performed up to 16 kHz and the numerical simulations on the 200-

12800 Hz frequency range. Results are discussed and compared in the next paragraph.

5 Results and discussions

Experimental map of radial velocities at the surface of the cylindrical shell are plotted for two frequencies on Figure 4. The frequencies are chosen so that the cylindrical shell exhibits particular features. At 1980 Hz, the global mode *(m,n)=(4,4)* can be seen (*m* being the longitudinal mode number and *n* the circumferential order). At 3580 Hz only the section close to the shaker is vibrating exhibiting a local mode. This is explained by the fact that the stiffeners spacing is different on the other sections of the cylinder and are not excited.

On the one hand, the experimental quadratic velocity averaged on all the points at the surface of the cylindrical shell is plotted as a function of the frequency on Figure 5 (red solid line). On the other hand, numerical calculations are performed with the CTF method. The calculations are done for 400 discrete frequencies and only take a few hours using Matlab on a computer with 64 Gb RAM. The black dashed curve in Figure 5 shows the results of the CTF method. The two curves show a good agreement. The differences seen are due to two main factors. In the low frequency range, the boundary conditions have a large effect on the response, and it is difficult to model properly end caps in the CTF method. One of the other limitations of the CTF method is that the structural damping is taken constant over the whole frequency range, which is not the case in reality. Here it seems that the structural damping is over-estimated in the high frequency range.

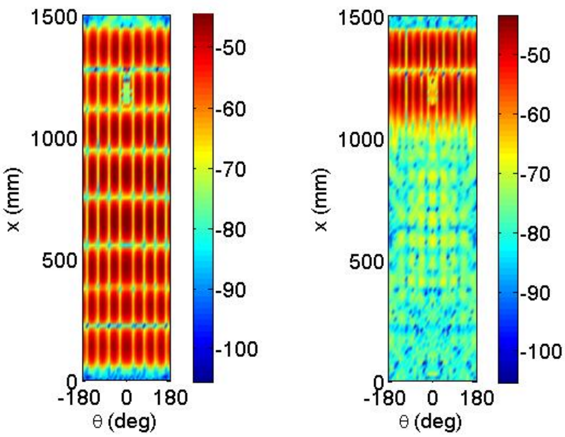

Fig. 4. Experimental map of radial velocities (dB ref 1 m.s-1) for f=1980 Hz (left) and f= 3580 Hz (right).

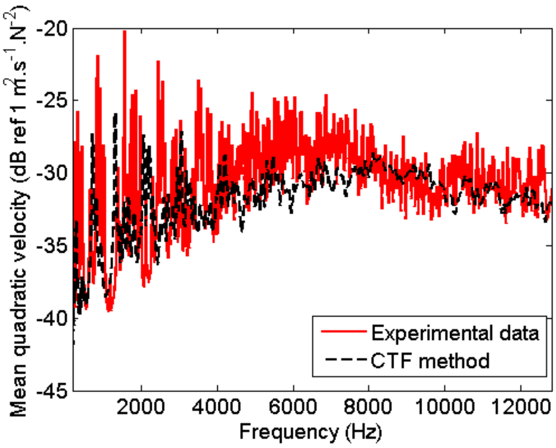

Fig. 5. Mean quadratic velocity of the cylindrical shell.

For each frequency, a Fourier transform and a Fourier series of the radial displacements at the surface of the shell is performed along respectively the axial and circumferential coordinates to yield the response in the so-called wavenumber domain. The wavenumber representation has interest for the vibro-acoustic analysis. An example of response in the wavenumber domain is given on Figure 6. Maxima of velocity can be seen at *(k_x,n)=(±17,4)*. The axial wavenumber k_x can be linked to the longitudinal mode number m by $k_x = \frac{2\pi m}{L}$. The white line shows the radiation circle. The velocity components inside this circle contribute strongly to the far-field radiated pressure [7].

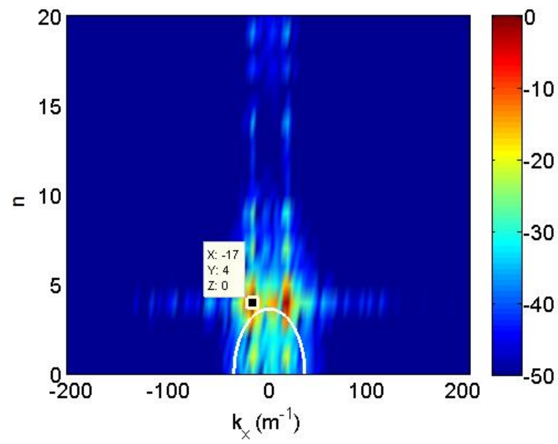

Fig. 6. Map of radial velocities (dB ref max) in the wavenumber domain at 1980 Hz.

The far-field radiated pressure at a distance R is calculated from the radial displacements in the wavenumber domain using the stationary phase theorem:

$$p(R,\varphi,\theta) = \sum_n \frac{2j\rho_0\omega^2}{Rk_0\cos\varphi}\frac{\widetilde{W}(-k_0\sin\varphi, n)}{H_n^{(2)\prime}(Rk_0\cos\varphi)} e^{-jRk_0 + jn(\theta + \frac{\pi}{2})}$$

with $\widetilde{W}$ the displacements in the wavenumber domain, $k_0 = \omega/c$ the acoustic wavenumber and $H_n^{(2)\prime}$ the derivative of the Hankel function of the second kind and order n. The radiated pressure around the cylindrical shell calculated for the frequency of f=1980 Hz is plotted in Figure 7. On this figure, the cylindrical shell is in a horizontal position. It can be seen that the extremities of the cylindrical shell radiate in a more efficient way than the middle part.

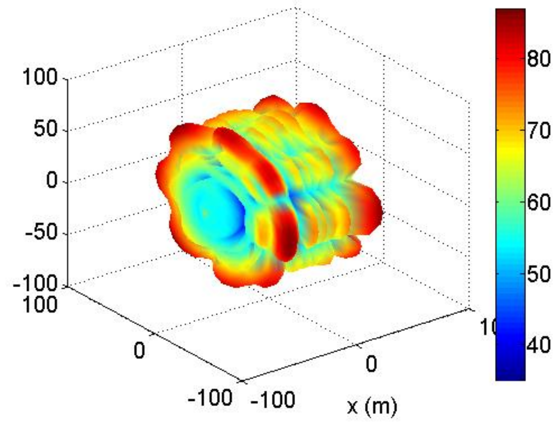

Fig. 7. Far-field radiated pressure (dB ref 2e-5 Pa) at f=1980 Hz.

The radiated power W_a is calculated by integrating the square of the pressure on an enclosed surface around the system. In the case of the radiated pressure calculated through the stationary phase theorem, it is calculated by the following integral:

$$W_a = \frac{1}{\rho_0 c_0} \int_{\varphi=0}^{\pi} \int_{\theta=0}^{2\pi} p^2(R,\varphi,\theta) R^2 \sin\theta d\theta d\varphi$$

In Figure 8, this quantity is plotted as a function of the frequency and compared to the power measured using the microphone array. The power measured with the microphones is only a rough estimation of the radiated power: the microphone spacing in the axial direction is too large, the microphones are not totally enclosing the surface around the cylindrical shell, the microphones are not in the far-field in the low frequency range and there are reflections on the floor. The numerical results calculated with the CTF method are also shown. The same comments than for Figure 5 can be made: differences can be seen at low frequencies because of the boundary conditions, and at high frequencies because of the structural damping. However, although all these different assumptions, it can be said that there is a good agreement between the three curves.

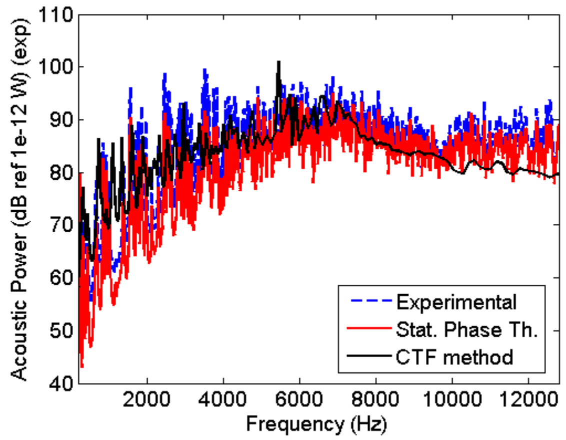

Fig. 8. Acoustic radiated power (dB ref 1e-12 W) by the cylindrical shell as a function of the frequency.

6 Conclusions

Two methods, one numerical and the other experimental, have been presented for the analysis of the noise and vibration of a stiffened cylindrical shell. They have been applied to an academic test case in air. The comparison of the numerical and experimental results highlights the importance of the boundary conditions in the low frequencies and of the structural damping in the high frequencies. However, these comparisons show that the numerical method developed here is a powerful tool to assess the noise and vibration performances of industrial systems. More particularly, strong coupling with water can be taken into account without dramatically increasing the calculation cost.

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Valentin Meyer received his engineering degree at ENSTA ParisTech in 2013. He received his PhD at INSA Lyon in 2016 on the topic of vibro-acoustic modeling of immersed structures, in collaboration with Naval Group. Since 2016 he works as a R&D engineer in acoustics and vibrations at Naval Group Research.

Laurent Maxit
Laurent Maxit holds a PhD in acoustics in 2000. He worked as research engineer for the French ministry of defence before joining INSA Lyon in 2008 as associate professor. His main research topic consists in the developments of advanced modelling methods dedicated to the mid to the high frequency ranges.