Delay-Doppler Waveforms for Underwater Acoustic Communications

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Technical Paper. Reliable underwater acoustic (UWA) communication is limited by long multipath delays, severe Doppler effects including non-uniform shifts across the signal bandwidth, constrained bandwidth at lower communications frequencies, and acoustic noise. These effects are more pronounced in underwater mobility scenarios at kilometre-scale communications distances. This paper examines the conditions under which the delay-Doppler (DD) domain representation, already shown to mitigate the effects of delay and Doppler spreading in RF channels, provides a similarly useful model for UWA propagation. Subsequently, Orthogonal Frequency Division Multiplexing (OFDM) and Orthogonal Time Frequency Space (OTFS) modulation schemes are compared as potential candidate for such channels. A simple model implementing a multipath UWA channel that incorporates wideband Doppler scaling effects is introduced. Simulation results demonstrate that OTFS, through its ability to address mobility-induced Doppler shifts, achieves higher performance relative to OFDM as the relative velocities of objects and surfaces within the channel are increased.

1 Introduction

1.1 The Need for Reliable Underwater Acoustic (UWA) Communications

The use of underwater vehicles, distributed sensors and remote platforms is increasing due to evolving defense, environmental and commercial use cases. Simultaneously, requirements exist for increasingly secure and reliable communications links, with higher data rates for command, control, telemetry and payload communications. Acoustic, optical and radio links are candidate technologies for underwater communications [1]. However, for RF communications, signal bandwidths and communication distances are constrained due to the conductivity of seawater over typical frequencies. Similarly, dissipation, scattering and spreading effects limit achieved bandwidth and range over underwater optical channels.

An Underwater Acoustic (UWA) communications channel is a viable alternative, but it too presents unique characteristics and constraints compared to RF and optical approaches. Despite potentially increased communications distances, UWA is characterised by unique and challenging environment conditions that impact relibility, bandwidth and achieved data rates, including the following factors:

- Reflections from scatterers on the sea floor, at the surface, and within the channel create multipath *delay spread* that causes frequency selectivity and errors; and
- Platform, surface and sea floor motion introduces *Doppler spreading*, causing time selectivity and channel estimation challenges.

Single carrier (SC) modulation schemes have been studied in UWA communication scenarios, and shown to be effective [2]. However, the use of multicarrier (MC) modulation schemes are attractive, which are able to combat the effects of Inter-Symbol Interference (ISI) in frequency-selective channels. Consequently, several variants of Orthogonal Frequency Division Multiplexing (OFDM) have been proposed to improve performance in UWA channels characterised by delay spread [3, 4]. A limitation of OFDM-based modulation schemes is that they are not effective at mitigating the effects of Inter-Carrier Interference (ICI) in time-selective channels characterised by Doppler spread. Alternative Delay-Doppler communications schemes, such as Orthogonal Time Frequency Space (OTFS), have therefore been proposed for UWA channels [5]. OTFS utilises a simple, typically sparse channel model in the Delay-Doppler (DD) domain. Unlike OFDM, OTFS distributes data symbols across the time and frequency extent of the signal. Notwithstanding the unique channel impairments within UWA channels, OTFS has been conceived to simultaneously address ICI and ISI to improve reliability and performance of doubly-spread channels [6].

1.2 Scope and Contributions of this Paper

In the following section, a review of the UWA channel will be presented. Unique factors are identified that impact the reliability and performance of UWA communication.

OFDM is introduced in Section 3, a MC modulation scheme that mitigates the effects of ISI in frequency selective channels characterised by delay spread. OTFS, an alternate MC scheme based on modulation in the DD domain, is reviewed in Section 4. OTFS mechanisms that help combat ISI and ICI concurrently are described, which can improve performance in doubly-selective channels characterised by both delay-spread and Doppler spread.

Section 5 presents a simplified channel simulation model, using OFDM and OTFS modulation, that incorporates the effects of Doppler scaling typical of wideband UWA channels. Unlike an RF channel, OTFS modulation in a UWA channel cannot completely overcome the effects of ICI due to the Doppler scaling effect. The operating regimes where OTFS improves performance relative to OFDM are identified. It is shown that OTFS modulation, with its support for ICI mitigation, reduces symbol errors as the relative mobility (Doppler spread) in the UWA channel is increased. Section 6 concludes with a brief description of future work.

2 The UWA Communications Channel

2.1 Challenges

The UWA channel presents a number of challenges to the achievement of reliable, performant communications [7].

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These include the following:

- frequency-dependent path loss, or attenuation that is variable over the signal bandwidth;
- uncertain channel geometry, where locations of senders, receivers and scattering surfaces are effectively random;
- channel parameters that vary according to depth, distance from shore, temperature and other factors [8];
- communications paths that are not typically straight, but rather curves described by Snell's law, and where signals may arrive more quickly over longer paths due to the varying speed of sound in water [8];
- a signal characteristic that is wideband relative to its transmission frequency;
- fading behaviour that is not adequately described by Ricean and Rayleigh models;
- sensitivity to external factors such as wind, waves and bubble concentrations that are difficult to model; and
- random Doppler shifts caused by changes in geometry or motion over different time periods.

Additionally, noise in UWA channels does not conform to an Additive White Gaussian Noise (AWGN) model typical of terrestrial RF channels. Underwater noise is typically not white, and is non-stationary. The channel is also impacted by other forms of impulsive noise, such as "shrimp snapping", wind noise, and for UWA channels close to the shoreline, human-generated construction noise [9].

2.2 Channel Description of UWA Channels with Doppler Scaling

The multipath UWA channel time-varying impulse response (TVIR) is given by:

$$h(\tau,t) = \sum_p h_p \delta(\tau - \tau_p(t)) \tag{1}$$

Here $\tau_p(t)$ is the path delay, which is time varying and incorporates normalised relative velocity a, where:

$$\tau_p(t) \approx \tau_p - at \tag{2}$$

Equation 2 assumes that there is single dominant relative velocity that is shared by all paths [3]. The signal received over each path is either compressed or dilated in time, depending on the direction of the relative velocity v_{rel} over that path:

$$t \rightarrow \frac{t}{(1+a)} \tag{3}$$

where $a = \frac{v_{rel}}{c}$ and c is the speed of sound in the channel. The Doppler scaling[1] parameter α is a significant factor in UWA channels characterised by low propagation speeds and low passband transmission frequencies. It is related to the normalised relative velocity a by (4):

$$\alpha = \frac{1+a}{1-a} \tag{4}$$

In [3], it is assumed that there is a single Doppler scaling factor within the channel, or alternatively a single dominant relative velocity between sender and receiver that is common to all paths within the channel. This is possible when the sender or receiver is attached to a vessel that is moving at a higher speed than other less mobile scatterers within the channel. However, in general this is not the case, for example when reflections occur against fast-moving surface waves or against moving underwater vessels. When multiple Doppler scaling factors or relative velocities occur within the channel, the less dominant paths will tend to cause interference against the dominant one. Simulations in Section 5 model the case where each path within the channel has a unique normalised relative velocity (a_p).

2.3 Mitigation of Doppler Scaling at the Receiver

Mitigating the effects of Doppler scaling occurs by estimation of the relative velocity a and resampling the received signal. The factor $\hat{a}$ is estimated in noise from a known preamble signal, often a Doppler-resilient, frequency-modulated "chirp".

In [3], the received signal $y(t)$ is resampled with the estimated $\hat{a}$ parameter:

$$z(t) = y\left(\frac{t}{1+\hat{a}}\right) \tag{5}$$

so that:

$$\frac{t}{(1+a)} \rightarrow \frac{(1+\hat{a})}{(1+a)} t \approx t \tag{6}$$

After resampling, the residual Doppler shift ε is given by:

$$\varepsilon = \left(\frac{a-\hat{a}}{1+\hat{a}}\right) f_c \tag{7}$$

Assuming $\hat{a}$ is an accurate estimate of a, the residual ε will be small, and analogous to a Carrier Frequency Offset (CFO) at the receiver. That is, ε represents a frequency-independent Doppler *shift* across the entire signal bandwidth. After estimation, $\hat{\varepsilon}$ can be removed by a phase rotation of the resampled signal.

2.4 Comparison of UWA to RF Terrestrial Channel

In Section 2.1, the UWA channel was described as wideband in nature, due to a signal bandwidth B which is comparable in magnitude to the passband center frequency f_c. This is not typical of narrowband RF terrestrial signals, which typically have a passband center frequency that is a few orders of magnitude greater than the signal bandwidth. For the narrowband case, relative motion between sender and receiver causes a Doppler shift that is *approximately* frequency-independent across the signal bandwidth. Considering a channel comprising a single path with unity gain and zero delay, the effect of the Doppler shift ν (Hz) on the transmitted signal $x(t)$ is a phase rotation of the transmitted signal, given by:

$$x(t)e^{j2\pi\nu t} \tag{8}$$

As described in Section 2.3, for the wideband UWA channel, the Doppler scaling is observed as time compression or dilation. This is *not* equivalent to a constant frequency phase rotation, as higher frequency components within the signal incur a higher shift than the lower frequency components.

[1] Doppler Scaling is alternatively called the "Doppler Squint Effect" (DSE) in [10], anagolous to the "beam squint" effect introduced by the phase rotation of wideband signals in phased array beamforming scenarios.

Traditional equalization schemes, such as LMMSE in OTFS, assume a narrowband signal. They can therefore compensate for channel Doppler shifts with equalization schemes that implement simple phase rotation rather than more complex resampling. To achieve high performance and minimise errors in UWA channels, using traditional equalization schemes will require resampling to remove at least the dominant Doppler scaling component from the signal as a prerequisite to further narrowband signal processing.

3 Multicarrier Modulation in the Time-Frequency Domain

3.1 Orthogonal Frequency Division Multiplexing (OFDM)

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier (MC) modulation scheme for frequency-selective channels that are characterised by delay spread. Mitigation of frequency-selective fading across the signal bandwidth is achieved by dividing the signal into multiple orthogonal spectral components, each of which has an effectively flat-fading characteristic over the narrower subcarrier bandwidth that can be more easily equalized.

OFDM modulation is implemented by an N-point Inverse Discrete Fourier Transform (IDFT) of symbol sequence $s_0, \ldots, s_{N-1}$, followed by a digital-to-analog conversion of the IDFT data, as shown in (9):

$$s(t) = \sum_{k=0}^{N-1} s_k e^{j2\pi(f_0+\Delta f)t} \tag{9}$$

where $0 < t < T$, and $\Delta f = 1/T$ for orthogonality.

Padding is generally added between OFDM symbols to address Inter-Symbol Interference (ISI). Optimally, the time extent of the padding must be at least as large as the delay spread, $\Delta\tau$, of the channel. In contrast to zero padding, OFDM implementations alternatively utilise a cyclic prefix (CP), where the final portion of the symbol data is copied and attached to the front. In addition to addressing ISI, the CP effectively converts the channel's time-domain response from a linear convolution to a cyclic convolution over the OFDM symbol time (T). This allows the channel to be equivalently represented by multiplication in the frequency domain, preserves subcarrier orthogonality, and facilitates the use of a simple, per-subcarrier, single tap equalization scheme, implemented in the frequency domain.

Although enhancements have been proposed to address its time-varying behaviour [11], traditionally OFDM has not provided a mechanism to address the ICI caused by Doppler spread within the channel. Alternatively, DD modulation schemes such as OTFS, as described in the following section, provide the ability to address both time- and frequency-selectivity in doubly spread channels concurrently.

3.2 OFDM in UWA Channels

The use of OFDM in UWA channels is well-established, due to the multipath characteristic of such channels and the resulting frequency-selective fading. A Zero Padding (ZP) OFDM implementation for UWA is described in [3]. A more advanced OFDM implementation, implementing receiver feedback-based adaptive encoding and modulation techniques to optimise power and modulation levels of subcarriers, is described in [4]. Both of these implementations consider the Doppler scaling behaviour introduced in UWA channels, as described in Section 2.2.

4 Delay-Doppler Communications

4.1 Motivation

In the previous section, OFDM was presented as an MC modulation scheme capable of mitigating the effects of frequency selectivity and ISI caused by multipath delay spread within the channel. However, OFDM has no intrinsic mechnanism to address time selectivity and ICI caused by Doppler spread. Delay-Doppler modulation and channel estimation schemes have been proposed to address ISI and ICI simultaneously in doubly-selective channels.

4.2 Delay-Doppler (DD) Channel Description

A DD representation characterizes the channel using a two-dimensional grid of delay (τ) and Doppler (ν) parameters. These parameters relate to physical properties (e.g positions and velocities) of scatterers within the channel, and hence the components of the DD representation reflect the physical environment. As shown in Figure 1, a sparse collection of paths, reflecting a limited number of scatterers, are represented in localized regions of the DD plane.

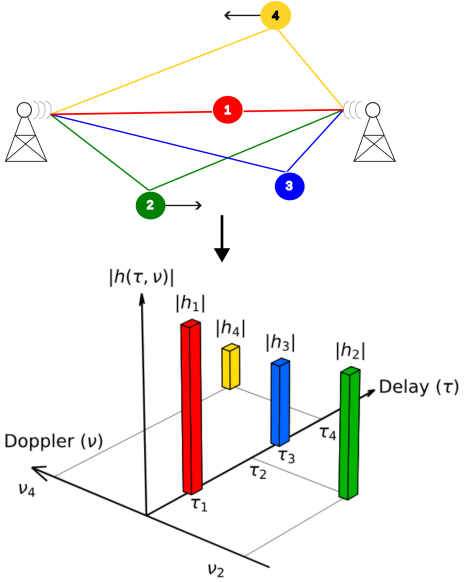


Figure 1: Multiple Paths and DD Channel Representation

A multipath channel can be represented in the DD domain via its DD spreading function, defined in (10):

$$h(\tau,\nu) = \sum_p h_p \delta(\tau-\tau_p)\delta(\nu-\nu_p) \tag{10}$$

Each path is parameterised by its delay τ_p, Doppler ν_p and complex gain h_p variables [9].

The Input-Output relation for DD communications channels is given by (11):

$$y(t) = \iint h(\tau,\nu)x(t-\tau)e^{j2\pi\nu(t-\tau)}\,d\tau\,d\nu \tag{11}$$

where $y(t)$ is the received signal, $x(t)$ is the transmitted signal, and $h(\tau,\nu)$ is the DD spreading function of the effective DD-domain channel filter.

4.3 Orthogonal Time Frequency Space (OTFS) Modulation

OTFS modulates information in the DD-domain, with information symbols placed on a discrete $M \times N$ grid. In the multicarrier variant of OTFS (MC-OTFS), the DD representation is converted to Time-Frequency (TF) representation through the Inverse Symplectic Finite Fourier Transform (ISFFT), and the time domain (TD) waveform is generated using Heisenberg Transform [12, 6]. At the receiver, the TD waveform is converted to the TF- and DD-domains via the Wigner Transform and Symplectic Finite Fourier Transform (SFFT) respectively.

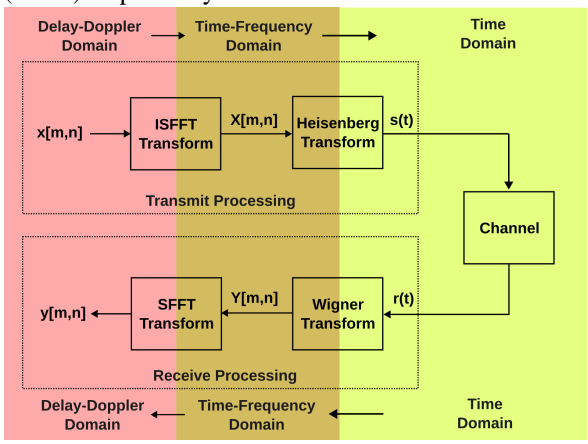


Figure 2: MC-OTFS Transmit and Receive Processing

The conversion of OTFS symbols $x[k,l]$ to TD domain representation is given by (12):

$$X[n,m] = SFFT^{-1}(x[k,l]) = \frac{1}{MN}\sum_{k=-N/2}^{N/2-1}\sum_{l=0}^{M-1} x[k,l]e^{j2\pi(\frac{nk}{N}-\frac{ml}{M})} \tag{12}$$

Each information symbol in the DD domain is spread across the full time-frequency extent of the signal. Each information symbol incurs constant gain through the effective channel, and OTFS equalizers operating in the DD domain can exploit the full diversity of independent paths in the channel. This is fundamentally different behaviour to OFDM, where non-primary fading paths place some of their energy into the padded regions of the signal, which are discarded at the receiver before the demodulation occurs.

As an alternative to MC-OTFS, a newer Zak-OTFS variant has been proposed [6]. Zak-OTFS supports single-step conversions to/from the TD using the Zak transform:

$$Z_x^{(L,K)}[n,k] \triangleq \frac{1}{\sqrt{K}}\sum_{l=0}^{K-1} x[n+lL]e^{-j2\pi\frac{k}{K}l}, \quad n,k \in \mathbb{Z} \tag{13}$$

In Zak-OTFS, signal processing is performed in the DD domain. Filtering operations are described by *twisted convolution* (TC) in the DD domain, denoted by the $*_\sigma$ symbol:

$$a(\tau,\nu) *_\sigma b(\tau,\nu) \triangleq \iint a(\tau',\nu')b(\tau-\tau',\nu-\nu')e^{j2\pi\nu'(\tau-\tau')}d\tau' d\nu' \tag{14}$$

Practical implementation of TC filters for Zak-OTFS have been proposed in [13, 14].

4.4 OTFS Channel Estimation and Equalisation in the DD Domain

As shown in Figure 3, the benefit of representing a doubly-spread channel in the DD domain is that a small number of dominant paths result in a sparse, quasi-stationary DD representation even when the corresponding TF channel representation changes rapidly in both time and frequency. This can reduce the number of parameters needed for channel estimation when performed in the DD domain and facilitates sparse equalisation techniques [15, 16, 17].

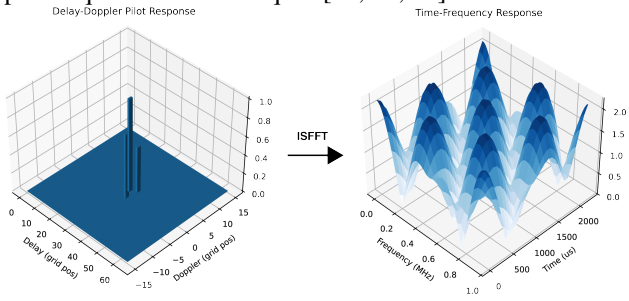


Figure 3: DD and TF Channel Representations

Channel estimation in OTFS is typically pilot-based, where a known pilot symbol is placed at a known location in the DD grid. As each information symbol in the OTFS frame incurs constant gain through the channel, the entire OTFS frame can be equalized from a single pilot symbol placed anywhere in the grid.

Linear Minimum Mean Square Error Estimation (LMMSE) is a common technique for equalisation of OTFS signals [18]. A block-circulant channel matrix is formed from dominant taps in the effective channel's response to the pilot symbol. The received DD signal is linearised and estimated. Alternate techniques, such as low-complexity message passing (MP) detectors have been proposed in [19].

4.5 UWA Channels in the DD Domain

UWA communications channels are doubly dispersive, and as such may benefit from representation in the DD domain when multiple paths and mobility within the channel cause path-specific delay and Doppler variation. Compared to equivalent RF channels, Doppler variations are exacerbated by the low speed of sound c in seawater and multiple reflective surfaces of varying delay. These factors motivate the examination of OTFS as a candidate modulation scheme for reliable, performant UWA communications.

The challenge in implementing OTFS in UWA channels is the wideband nature of the signal,which introduces the Doppler scaling described in Section 2.2. OTFS, however, incorporates a narrowband assumption. The DD spreading function in (10) has a per-path Doppler shift, ν_p, which is constant across the bandwidth of the signal. Resampling of the signal to remove the dominant Doppler spreading component is therefore required [10]. UWA channels might also be overspread, where the delay and Doppler spread exceeds the support of the DD grid. Resampling can remove this overspreading in certain scenarios [5].

An OTFS receiver chain that includes Doppler compensation is shown in Figure 4. OTFS signal processing blocks can exploit the channel's residual narrowband appearance after Doppler scaling compensation has been performed.

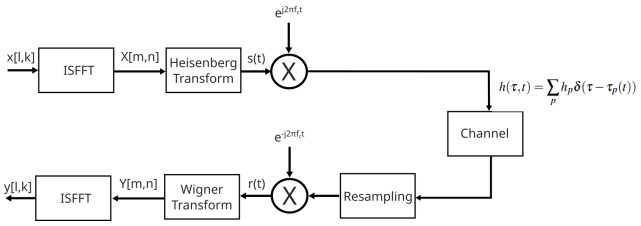


Figure 4: OTFS Processing Chain including Resampling

After resampling, if strong paths remain that contain channel objects with differing velocities, residual Doppler scaling may affect the signal.

Another issue arising when OTFS is used in UWA channels is *fractional Doppler shift*. The number of delay bins, M, is typically large and impacts the overall bandwidth of the signal ($M\Delta f$). However, the number of Doppler bins, N, is typically very small because the time extent of the signal (NT) needs to be minimised to maintain an accurate channel estimate. A consequence of the small N is that "leakage" of energy occurs between Doppler bins in the DD signal representation, reducing sparsity and potentially introducing channel estimation errors and equalization overhead.

5 A UWA Model and Simulation Results

5.1 Model Scenario

A simplified wideband UWA channel model has been created, as shown in Figure 5. This model has been designed to simulate sparse, multipath UWA channels, where scattering objects exist within the channel that have different relative velocities. This allows simulation of varying amounts of Doppler scaling, as well as the residual Doppler scaling that still exists after resampling has occurred at the receiver.

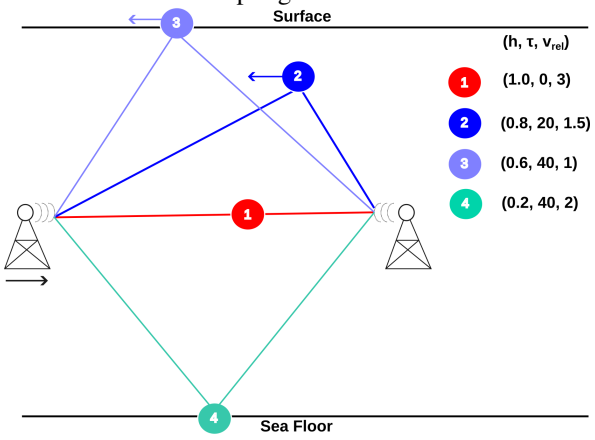


Figure 5: Simple Wideband UWA Communications Model

The model is based on time compression and dilation for each path within the channel. As such, it is fundamentally different to models used for RF simulation that have constant Doppler *shifts* across the signal bandwidth. The receiver performs a resampling operation using the actual normalised relative velocity (a) of the dominant path (path 1). However, the resampled signal will still contain the Doppler scaling associated with the longer, weaker paths (path 2 and 3), which will not be fully resolvable by the traditional narrowband equalisation schemes that follow.

Simulation results compare the performance of the OFDM and OTFS schemes and determine the conditions under which each are optimal for UWA channels.

5.2 Simulation Parameters

Table 1 provides the simulation parameters for the model.

Table 1. Modulation Scheme Comparison - UWA Parameters

Quantity	Initial value
Acoustic band	10-18 kHz with 8-kHz occupied bandwidth
Geometry	1-2 km shallow-water link with about 100 m water depth [16]
Protocol Variants	MC-OTFS, CP-OFDM
OTFS grid	$M = 400$ and $N = 20$ with 20 Hz subcarrier spacing (Δf), 0.05 s symbol time (T)
Frame duration	$NT = 1s$, excl. padding
Padding Configuration	M samples Cyclic Prefix (OFDM), M samples Cyclic Prefix/Postfix (OTFS)
Modulation	16-QAM
Delay spread	Up to 35 ms in the baseline channel [16]
Relative velocity (max)	$v_{rel} = 5m/s$, corresponding to a normalised velocity of $a = 0.0033$
Channel knowledge	Perfect channel knowledge
Detection	LMMSE
Pulse Shaping	Raised Root Cosine ($r = 0.05$)
Noise model	AWGN
SNR range	0-30 dB

5.3 Results

Figure 6 shows the uncoded BER performance of the OTFS and OFDM schemes as the residual Doppler scaling α is varied, for a fixed SNR value of 15dB.

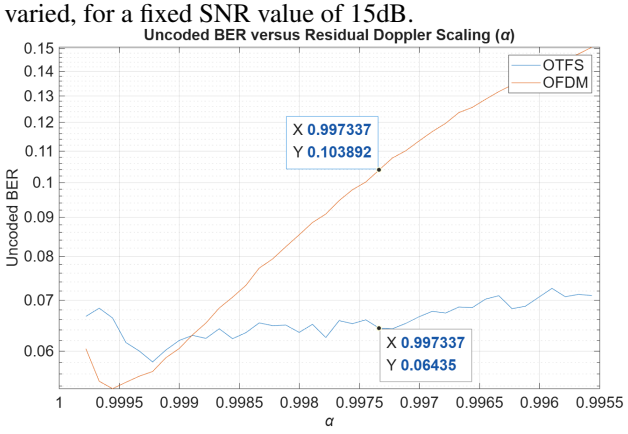


Figure 6: Uncoded BER versus Residual Doppler, 15dB SNR

Figure 7 shows the uncoded BER performance of the OTFS and OFDM schemes as the SNR is varied, for a fixed residual Doppler scaling $\alpha = 0.9973$ ($v = 2$ m/s).

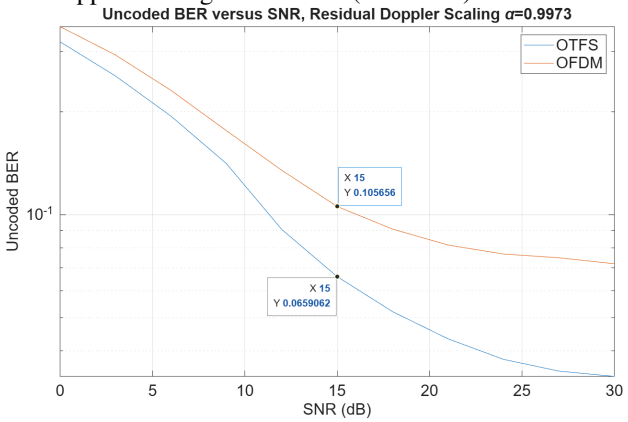


Figure 7: Uncoded BER versus SNR, $\alpha = 0.9973$

Note that the labelled points in the two Figures correspond to the same operating point in the model.
The following results can be observed from Figure 6 and Figure 7:

- The performance of OTFS and OFDM is similar for small residual Doppler scaling ($0.9989 < \alpha < 1$, or $v_{rel} < 0.83$ m/s). This is because after the dominant Doppler scaling component is removed, the residual Doppler scaling causes minimal ICI.
- The uncoded BER performance of OFDM degrades as the residual Doppler scaling α is increased. This is due to unresolvable residual ICI after resampling.
- OTFS has a fairly constant performance even as the residual Doppler scaling is increased. This is due to the narrowband equalisation mechanism in OTFS that can mitigate Doppler shifts and resolve ICI.
- At the highest Doppler scaling (or relative velocity) values, OTFS significantly outperforms OFDM, with approximately twice the BER performance of OFDM.
- For a specific residual Doppler scaling value ($\alpha = 0.9973$), the performance advantage of OTFS over OFDM is maintained across the range of tested SNR values.
- The uncoded BER values are still relatively high across all tested scenarios. This is because OTFS contains a mechanism to mitigate narrowband Doppler shifts, but not wideband Doppler scaling. OFDM contains no mechanism to address either problem.

6 Conclusions and Future Work

This paper has detailed the characteristics and challenges associated with wideband UWA communications channels. The time compression and dilation caused by Doppler scaling, specific to wideband channels, has been described, along with a mitigation mechanism achieved by signal resampling at the receiver.
A simulation model has been developed to compare the performance of OFDM and OTFS modulation schemes in wideband UWA channels. It has been shown that as the residual Doppler scaling after resampling is increased ($\alpha < 0.9989$ or $v_{rel} > 0.83$ m/s), OTFS outperforms OFDM due to its inclusion of an ICI mechanism that can mitigate narrowband Doppler shifts. However, neither OFDM or OTFS are ideal for wideband UWA communications scenarios, due to the lack of a mechanism that fully resolves the residual Doppler scaling that remains after signal resampling.
In the future, the simulation model shall be expanded to address more of the UWA communications challenges listed in Section 2.1.

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