GEOMETRICAL INVERSION COUPLED WITH AUTOMATED GEOLOGICAL MODELLING

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Summary

We introduce and test a method that integrates automatic geological modelling in level-set inversion. The objective of this method is to encourage geological realism during deterministic geophysical inversion for the modelling of interfaces between rock units. To this end, we introduce a geological correction term in the model update to reduce geological inconsistencies at each iteration of the inversion. This is achieved thanks to the integration of an automatic implicit geological modelling scheme within the geophysical inversion algorithm. After introducing the main theoretical aspect of the approach we propose, we present a realistic synthetic case illustrating the proof-of-concept in a basin scenario using gravity data. Results indicate that our approach effectively steers inversion towards geologically consistent models and may therefore be applied to field data.

Geometrical inversion coupled with automated geological modelling

Introduction

One of the challenges faced by potential field data inversion is the recovery of geologically realistic models. This is due, in part, to the strong non-uniqueness of the solution, which allows a large number of models to fit a given gravity or magnetic survey while remaining geologically inconsistent or ambiguous to interpret. This limitation has called for the development of a variety of constraints to reduce the range of physically admissible models and to steer inversion towards geologically reasonable outcomes (Moorkamp et al., 2016; Wellmann and Caumon, 2018). A promising strategy that has been investigated in recent years is the use of level-set inversions to invert directly for the geometry of geological bodies. In the context of exploration geophysics, recent works have used level-set inversion in for the recovery of one or two anomalous geological units, be it for single physics or joint inversion (Li et al., 2016, 2017; Zheglova et al., 2018) before extension to an arbitrary number of rock units by Giraud et al. (2021) in the case of 3D gravity inversion. Compared to the more common smooth, continuous inversion approaches, these present a progress towards recovering geologically reasonable models from potential field data. Nevertheless, to the best of our knowledge, existing level-set inversion schemes lack rigorous geological control. An obvious solution to this problem is to enforce structural geological constraints in the formulation of the level-set inverse problem, which is the object of the method proposed here. We present an inversion algorithm building up on the generalized level-set inversion approach of Giraud et al. (2021) and Rashidifard et al. (2021) that uses geological constraints by integrating the automatic implicit geological modelling engine LoopStructural (Grose et al., 2021) to the inversion process and integrates geological information in the definition of several elements of the objective function. In this abstract, we first introduce essential information allowing general understanding of the method. Following this, we present the proof-of-concept using 3D synthetic data corresponding to a stratigraphic, layered geology constrained by two 2D seismic lines, and stratigraphic rules applied in the whole area. Finally, we discuss the results and extensions of this work.

Methods and theory

The method we propose here is a extension of Giraud et al. (2021) to the utilisation of automatic geological modelling to ensure geological consistency of inverted models with a new formulation of the cost function. Like all level-set inversions, this approach considers a set of signed-distances $\boldsymbol{\phi} = \{\boldsymbol{\phi}_k, k = 1, \ldots, N\}$ to the interfaces between geological entities such as units or groups as the quantity inverted for, such that:

$$\boldsymbol{\phi}_k \begin{cases} > 0 \; inside \; entity \; k \\ = 0 \; at \; interfaces \\ < 0 \; outside \; entity \; k \end{cases} \tag{1}$$

It is calculated using the fast-marching method of Sethian (1996). From the signed-distance values $\boldsymbol{\phi}$, a transform is applied to obtain a physical property model (e.g., density values):

$$\boldsymbol{m}(\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_N) = \sum_{i=1}^{N} \prod_{j=1, j\neq i}^{N} \left(1 - H(\boldsymbol{\phi}_j)\right) V_i H(\boldsymbol{\phi}_i), \tag{2}$$

where $\boldsymbol{V} \in \mathbb{R}^N$ is a vector containing the physical property (e.g., rock density) value assigned to each of the N geological entities modelled; H is the smeared-out Heaviside function, which we calculate following Osher and Fedkiw (2003, p.15). At each iteration of the inversion, the physical property model is updated from the changes of $\boldsymbol{\phi}$ that lead to a reduction in geophysical data misfit.

We formulate the inverse problem using a least-squares framework, solving for the update of the signed-distance, $\delta\boldsymbol{\phi}$. We minimize the geophysical data residuals $\boldsymbol{r}$ through optimization of $\Psi(\delta\boldsymbol{\phi}, \boldsymbol{r})$:

$$\Psi(\delta\boldsymbol{\phi}, \boldsymbol{r}) = \left\|\boldsymbol{J}^{\phi}\delta\boldsymbol{\phi} - \boldsymbol{r}\right\|_2^2 + \lambda_s\left\|\boldsymbol{W}(\delta\boldsymbol{\phi} - \delta\boldsymbol{\phi}^{prior})\right\|_2^2, \tag{3}$$

where $\boldsymbol{J}^{\boldsymbol{\phi}}$ is the sensitivity matrix of the data misfit with respect to changes in signed distances (see Giraud et al., 2021, for details about its computation); λ_s is a trade-off parameter that can be determined manually or through, e.g., L-curve analysis; $\delta\boldsymbol{\phi}^{prior}$ is the signed distance update for the current $\boldsymbol{\phi}$ values to be equal a reference model $\boldsymbol{\phi}^{prior}$ obtained from prior information such as geological modelling and/or prior geophysical information; $\boldsymbol{W}$ is a diagonal covariance matrix controlling the strength of the constraints assigned to each model-cell, which can be set accordingly with prior geological or geophysical information. Note that while we invert for the distances to interfaces between rock units, eq. 3 is similar to the 'traditional' formulation of inverse problems considering continuous property inversion. The inversion scheme we propose is summarized in Figure 1; inversion stops when stopping criteria regarding data misfit value or recovered model changes are met.

In addition to incorporating a geological model in $\Psi(\delta\boldsymbol{\phi}, \boldsymbol{r})$ through $\boldsymbol{\phi}^{prior}$ and geological uncertainty information in $\boldsymbol{W}$, we maintain geological consistency in the recovered model $\boldsymbol{m}$ (eq. 2) at each iteration by applying a geological correction term to $\delta\boldsymbol{\phi}$ obtained from solving eq. 3. For the n^{th} iteration, we first calculate $\boldsymbol{\phi}_*^n$, the signed distances update obtained from solving eq. 3:

$$\boldsymbol{\phi}_*^n = \boldsymbol{\phi}^{n-1} + \delta\boldsymbol{\phi}, \tag{4}$$

which we use to calculate a geological correction term as follows:

$$\delta\boldsymbol{\phi}^{geol}(\boldsymbol{\phi}_*^n) = f^{geol}(\boldsymbol{\phi}_*^n, \boldsymbol{d}^{geol}) - \boldsymbol{\phi}_*^n. \tag{5}$$

In eq. 5, f^{geol} identifies the nature of contacts between rock units corresponding to $\boldsymbol{\phi}_*^n$. It calculates the closest geological model honouring these contacts together with geological data and knowledge encapsulated in $\boldsymbol{d}^{geol}$, which include orientation data, foliations, etc. In this way, $f^{geol}(\boldsymbol{\phi}_*^n, \boldsymbol{d}^{geol})$ returns a set of signed distance values which account for relationships between units (e.g., stratigraphic thickness), known locations between contacts (e.g., seismic interpretation, borehole data, surface observations) and orientation data. The automated calculation of the geological model is performed using the geological implicit modelling engine LoopStructural (Grose et al., 2021). Using $\delta\boldsymbol{\phi}^{geol}$, we update the signed distance $\boldsymbol{\phi}$ as follows:

$$\boldsymbol{\phi}^n = \boldsymbol{\phi}_*^n + \alpha\delta\boldsymbol{\phi}^{geol}(\boldsymbol{\phi}_*^n) = (1-\alpha)\boldsymbol{\phi}_*^{nj} + \alpha f^{geol}(\boldsymbol{\phi}_*^{nl}, \boldsymbol{d}^{geol}), \alpha \in [0, 1[. \tag{6}$$

$\delta\boldsymbol{\phi}^{geol}$ adjusts the signed distance update $\delta\boldsymbol{\phi}$ proposed by the optimisation of the geophysical cost function $\Psi(\delta\boldsymbol{\phi}, \boldsymbol{r})$, ensuring geological consistency of the proposed model. At each iteration, $\delta\boldsymbol{\phi}^{geol}$ is calculated automatically by the application of a geological operator f^{geol} so that it represents the signed-distance update satisfying geological requirements; α adjusts the importance given to the geological correction term.
As a consequence of equation 5, $\delta\boldsymbol{\phi}^{geol}(\boldsymbol{\phi}_*^n)$ is equal to 0 everywhere the proposed geophysical update $\delta\boldsymbol{\phi}$ does not conflict with geological modelling, and differs otherwise, thereby steering inversion towards the region of the geophysical model space corresponding to geologically consistent models. In what follows, we set $\alpha = 1/2$ arbitrarily to give equal importance to $\delta\boldsymbol{\phi}$ and $\delta\boldsymbol{\phi}^{geol}$.

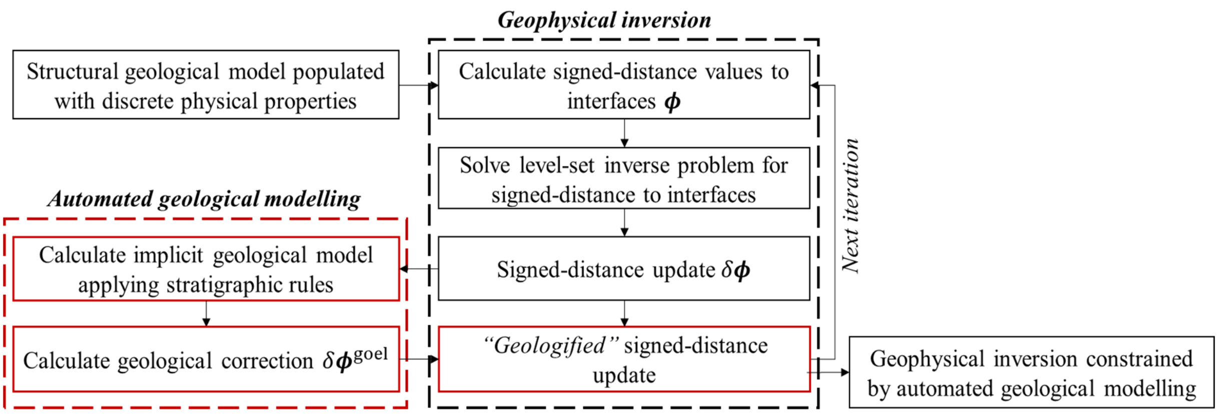

***Figure 1** Summary of the inversion scheme. The red boxes indicate in-inversion geological components of the algorithm. The red dashed line highlights automated geological modelling.*

Proof-of-concept

We generate the reference geological structural model starting from the Claudius dataset in the Carnavon Basin, offshore NW Australia (interpreted from WesternGeco seismic data made available by Geoscience Australia), which is freely available as part of the LoopStructural package (Grose et al., 2021). The original model is constituted of carbonate mounds overlaid by growth strata (see Figure 2d). We assume that two 2D seismic sections provide information to derive $\boldsymbol{\phi}^{prior}$; we set $\boldsymbol{W}$ so that that its values decay with distance to high-confidence areas. For the purpose of our testing, we modified the original geological data with the manual addition of a 500 m uplift as pointed by the red arrow. In what follows, we assess the capability of inversion to recover this feature of interest. We use a starting model where the uplift is missing, generating a strong data misfit. This leads the inversion to produce geologically unrealistic structures when no geological correction is applied (Figure 2b). On the contrary, visual inspection suggests that geological realism is preserved when geological correction is applied during inversion (Figure 2c). Coupled with the requirement of inversion to reduce data misfit, results obtained in this fashion are geologically and geophysically consistent and visually close to the reference model. While the two inversions discussed here present similar data misfit values, the recovered geological representation and the reference model may be different. To quantify the misfit between the recovered and reference model, we identify the contacts between rock units and record the number of model cells with a contact between rock units in the corresponding adjacency matrices. We use the difference between the adjacency matrices of the reference model and inversion results to evaluate the misfit of the different rock units modelled (Figure 2e). This supports our interpretation that the geological correction term allows inversion to produce models that are more geologically reasonable. For instance, contacts between unit 3 and 5 are precluded by the geological knowledge infused in inversion, and are not observed in the geologically corrected case.

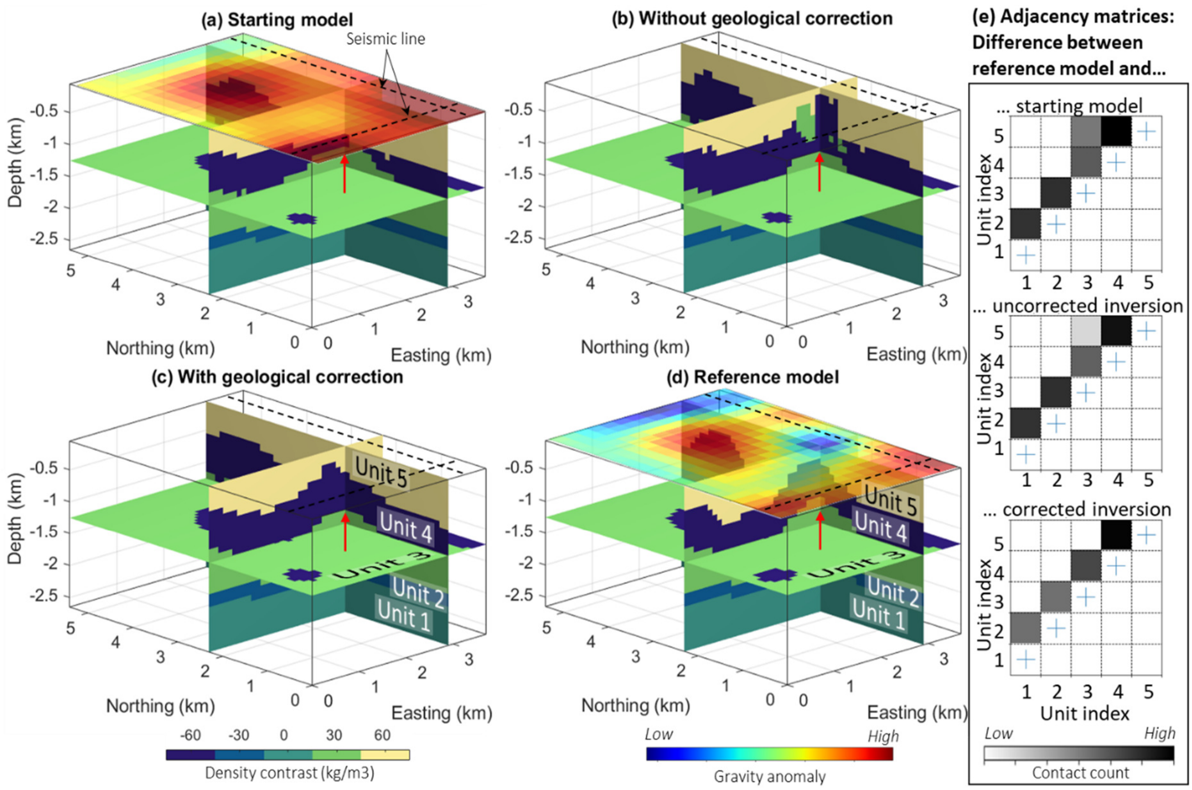

__Figure 2__ Synthetic test for proof-of-concept testing. The starting model is shown in (a), with the corresponding gravity anomaly shown in transparency; (b) and (c) show inversion results corresponding to geologically-free and corrected inversions, respectively; (d) shows the reference model and the corresponding gravity anomaly shown in transparency; (e) represents the difference between adjacency matrices for models shown in (a), (b) and (c) with the reference model (d). The location of the seismic sections used to derive the prior model is shown by the dashed lines.

Discussion and conclusions

We have presented and tested a method using geological rules to constrain level-set inversion and shown its efficacy through a synthetic proof-of-concept. From our results, we conclude that it can be applied to real world scenarios. The method also opens up avenues towards joint inversion of geological and geophysical data such as the exploration of the joint geology-geophysics model space, noting that inversions can run on a laptop computer in less than a minute. Work to be carried out in the near future includes testing multiple hypotheses. The workflow is highly flexible and allows the utilisation of a of prior information such as borehole data, seismic modelling, etc. Besides, we have not investigated in depth the use of $\boldsymbol{\phi}^{prior} = f^{geol}(\boldsymbol{\phi}_*^n, \boldsymbol{d}^{geol})$ in the cost function to re-calculate $\boldsymbol{\phi}^{prior}$ at each iteration, which is the object of future work. Lastly, a direct application of the proposed method is to ensure geological realism of rock unit models recovered ad hoc from geophysical inversion results.

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