JOINT INVERSION OF GRAVITY AND MAGNETIC FIELDS: FIRST RESULTS OF THE XORN PROJECT

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Summary

In the current work we present an innovative algorithm aiming at performing complete 3D joint inversion of gravity and magnetic fields properly constrained by interpreted seismic data and geological a-priori qualitative information.
Among the main advantages of the proposed inversion, there is the fact that it would allow to estimate at the same time geometries of the main geological features and 3D distributions of density and susceptibility of the studied volume. Moreover the inversion will keep sharp variations of density and susceptibility. The proposed algorithm is tested on a simple synthetic test case obtaining promising results.

Joint inversion of gravity and magnetic fields: first results of the XORN project

Introduction

Potential field methods, which exploit gravity and magnetic fields, are very powerful to recover fundamental information on the Earth's crust structure. Thanks to several dedicated satellite missions the major large-scale crustal features are now homogeneously depicted at high resolution. At regional and local levels instead a great variety of gravity and magnetic data have been collected at near surface altitudes in most regions of the world. One challenge to be faced today to fully exploit these global homogeneous satellite data and to merge them with regional datasets compiled at the Earth's surface is the development of ad-hoc processing techniques. Satellite and near surface measurements are complementary and are not sensitive to the same geological structures at depths. Inferring the Earth's crust structure, depicting the boundary of the geological units at depth and to a first order the stratification of the crust requires physics integrated approaches reconciling all measurements. Currently, the different sources of information when analysed individually suffer from non-uniqueness. Magnetic and gravity signals detect different crustal parameters and rarely coincide because various combinations of geological structures generate similar observations outside the sources (see e.g. Sampietro and Sanso (2012)). A promising solution is represented by the joint processing of both gravity and magnetic fields data, possibly incorporating the available geological knowledge and constraints coming from seismic acquisitions, in such a way to reduce the space of possible solutions. In the current work we will present an innovative, fully integrated approach, to perform a complete 3D joint inversion of gravity and magnetic fields data, properly constrained by seismic and geological a-priori information, developed in the framework of the XORN project funded by the European Space Agency within the EO4society Programme. The developed algorithm will be used within the project to recover a 3D regional model of the Earth crust in the Mediterranean Area in terms of density and magnetic susceptibility distribution within the volume, and in terms of depths of the main geological horizons separating the principal units (e.g. sediments, crystalline crust, mantle, etc.). The Mediterranean Area is in fact crucial to better understand the African-Eurasian active plates tectonics but also to lead industrial activities toward an optimal exploitation of natural resources.

Method and Theory

The proposed inversion method is based on a Bayesian assumption expressed by the Bayes Theorem that, in its general formulation, aims at retrieving the probability of a certain hypothesis given the observed evidence. We will start here from the Bayesian approach presented in Marchetti et al. (2019), and tested with optimal results in several real and synthetic case studies (e.g. Sampietro and Capponi (2021a,b); Sampietro et al. (2021)). The workflow to exploit this approach consists in collecting all the available information about the study area (in terms of geological horizons, average densities and their accuracies), to build an a-priori geological probabilistic 3D model (starting point of the inversion) and then choosing the solution that minimizes the residuals with respect to the observed gravity and, at the same time, maximizes the probability of the a-priori model. In other words, the inversion procedure modifies the initial model accordingly to the constrains imposed by the accuracy of the dataset used to build the a-priori model itself, trying to fit the gravity observations. From the mathematical point of view, the gravity inversion algorithm works on a volume discretized in volumetric elements (voxels). Each voxel i is described by a couple of random variables ρ_i (continuous variable expressing the density) and L_i (categorical variable characterizing the geological material, e.g. Plio-Quaternary sediments, Messinian salt, crust, etc., from now on called label). To estimate the refined model, the algorithm searches, by means of a Markov chain Monte Carlo method, the set of densities $\boldsymbol{\rho} = \{\rho_1, \rho_2, ..., \rho_N\}$ and labels $\boldsymbol{L} = \{L_1, L_2, ..., L_N\}$ maximizing the posterior probability given by the following function:

$$\phi = \left\{ \frac{1}{2}(\Delta g^o - \mathrm{F}\rho)^T \mathrm{C}_{\Delta g}^{-1}(\Delta g^o - \mathrm{F}\rho) + \frac{1}{2}(\rho - \overline{\rho}_\ell)^T \mathrm{C}_\ell^{-1}(\rho - \overline{\rho}_\ell) + \frac{1}{2}\gamma \sum_{i=1}^{N} s^2(L_i) + \right.$$
$$\left. + \frac{1}{2}\lambda \sum_{i=1}^{N} \sum_{j \in \Delta_i} q^2(L_i, L_j) \right\} \prod_{i=1}^{N} \chi_{[\overline{\rho}_\ell | 3\sigma_\ell^2]}(\rho_i) \quad (1)$$

where F is the forward operator to compute the gravitational field of each voxel of the model, $\overline{\rho}_\ell$ is the vector containing the average densities of the different labels ℓ, C_ℓ is a diagonal matrix containing the variances of the density of a given label ℓ, s^2 and q^2 are empirical functions defining the "distance" from the initial model and the "smoothness" of a possible solution, respectively.

Within this contest, the extension of the algorithm to the magnetic field is straightforward: it is in fact enough to substitute the gravity and density terms in Eq. 1 with two analogous terms based on the observed magnetic field (H^o) and distance from the average property generating the magnetic field (i.e. the magnetic susceptibility χ) thus obtaining a 3D magnetic only inversion.

$$\phi = \left\{ \frac{1}{2}(H^o - \mathrm{G}\chi)^T \mathrm{C}_{H}^{-1}(H^o - \mathrm{G}\chi) + \frac{1}{2}(\chi - \overline{\chi}_\ell)^T \mathrm{C}_{H\ell}^{-1}(\chi - \overline{\chi}_\ell) + \frac{1}{2}\gamma \sum_{i=1}^{N} s^2(L_i) + \right.$$
$$\left. + \frac{1}{2}\lambda \sum_{i=1}^{N} \sum_{j \in \Delta_i} q^2(L_i, L_j) \right\} \prod_{i=1}^{N} \chi_{[\overline{\rho}_\ell | 3\sigma_\ell^2]}(\rho_i) \quad (2)$$

or to combine Eq. 1 and Eq. 2 to obtain a fully integrated joint gravity and magnetic inversion:

$$\phi = \left\{ \frac{1}{2}(\Delta g^o - \mathrm{F}\rho)^T \mathrm{C}_{\Delta g}^{-1}(\Delta g^o - \mathrm{F}\rho) + \frac{1}{2}(\rho - \overline{\rho}_\ell)^T \mathrm{C}_\ell^{-1}(\rho - \overline{\rho}_\ell) + \right.$$
$$\frac{1}{2}(H^o - \mathrm{G}\chi)^T \mathrm{C}_{H}^{-1}(H^o - \mathrm{G}\chi) + \frac{1}{2}(\chi - \overline{\chi}_\ell)^T \mathrm{C}_{H\ell}^{-1}(\chi - \overline{\chi}_\ell) + \frac{1}{2}\gamma \sum_{i=1}^{N} s^2(L_i) +$$
$$\left. + \frac{1}{2}\lambda \sum_{i=1}^{N} \sum_{j \in \Delta_i} q^2(L_i, L_j) \right\} \prod_{i=1}^{N} \chi_{[\overline{\rho}_\ell | 3\sigma_\ell^2]}(\rho_i) \quad (3)$$

Note that in Eq. 3 the implicit relation between density and magnetic susceptibility of each voxel is given by the label, which defined for both the observables the a-priori range of variability of each geological layer.

Example on a Synthetic Case Study

Preliminary results have been obtained on a closed-loop simplified synthetic case study simulating the main geological characteristics of the Oka Carbonate Complex in Quebec, Canada. Treiman and Essene (1985); Gold et al. (1967); Thomas et al. (2016). The complex is made by two close intrusive centres resembling a distorted figure 8 shape in plan view. Each centre has a central plug of carbonatite surrounded by alkalic silicate rocks and an aureole of fenitization of the gneissic country rocks. The simplified scheme adopted to build the synthetic model is made by three main units defining the intrusive complex and a gneiss background with lateral density variations see Fig 1. In the figure also

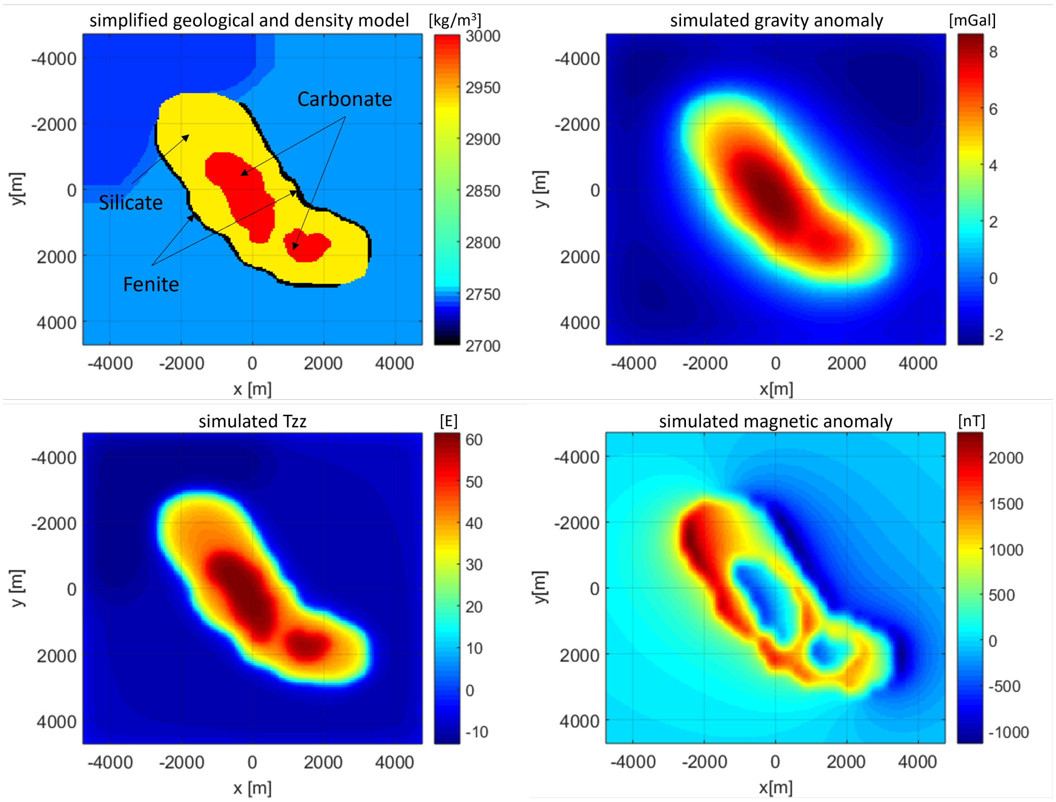

***Figure 1** Simplified geological model showing the density of the considered synthetic data; simulated gravity effect of the Oka complex in terms of gravity anomalies and second radial derivative of the gravitational potential and in terms of total magnetic intensity*

the simulated density values are presented. In order to simulate the magnetic field a magnetic susceptibility of 0.3 SI has been assigned to the annular silicate formation as the only magnetic unit within the complex. Starting from this initial model, and adding a correlated observation noise of 0.5 mGal, 1 E and 10 nT gravity and magnetic fields have been simulated. In particular, for the gravity field two functionals namely gravity anomalies (δg) and the second radial derivative of the potential (T_{zz}) have been simulated, while for the magnetic field only the total magnetic anomaly has been considered. The a-priori information has been obtained by modifying the true model in the geometries, density distribution and magnetic susceptibility. After applying the proposed inversion algorithm we notice that the the misfit with respect to gravity and magnetic has been reduced to values coherent with the simulated fields accuracy. The obtained solution, which has been obtained by modifying at the same time the 3D density and susceptibility distribution of each geological unit and the geometries of the layers show a better general agreement with respect to the starting approximated model.

Conclusions

In the current work we presented a procedure to perform a joint gravimetric magnetic inversion constrained by a-priori geological knowledge. The proposed approach, developed in the ESA XORN project founded in the framework of the EO4society Programme, is based on a Bayesian assumption and has already been extensively tested for gravity only inversion. The preliminary results, obtained for a simple synthetic case study are promising, since the inversion has been able to fit the potential fields observations and to improve the initial guessed model, thus fostering research and development activities to enhance the proposed algorithm for real-life applications.

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